# Horizon Europe Civil Security for Society 2026

***Disaster-Resilient Society for Europe***

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## Suggested contribution to *CL3-2026-01-DRS-04* (alternatively, contribution to *DRS-01* or *DRS-05*, given the necessary specifications)

*"Innovative Data-Based Quantitative Methodology of Disaster Prevention Planning and Management: Proof of Concept, Development, and Test of Decision Tools and Strategies"*

## Scope:

- Contribution intended to promote the identification and adoption of high readiness-level solutions by public and private sector organizations involved in disaster risk management, developed for scaling and commercializing innovative DRS measures, technologies and strategies to assure they reach end-uses efficiently.
- Promoted visibility of civil security research results.
- Applied research **continued from a recent publication** (https://doi.org/10.1007/s10479-024-06259-z), with application examples in Section 5 () and brief characterizations of the possible domains of application in Section 6 ("Disaster risk management") of that paper.
- Applications in close collaboration with end-users

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## About TUM

See https://www.tum.de/en/

## Desired Role

- Work package leader
- Task leader
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ORIGINAL RESEARCH

# Catastrophic risk: indication, quantitative assessment and management of rare extreme events using a non-expected utility framework

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**Abstract**
The paper develops a conceptual framework for the analysis and management of catastrophic risk. The framework serves to assess rare extreme events in systematic, quantitative and consistent ways. It dispenses with probabilistic extreme value theory, concentrating on descriptive statistics and simple probability distributions. Risk assessment is based on a recently developed axiomatic approach to non-expected utility preferences defined on the set of risky alternative courses of action available to an agent. The utility values of catastrophic risks are given an explicit algebraic representation, which shows them to be highly unstable ("elastic") in the sense that they respond disproportionately to small perturbations of the decision outcomes and their probabilities. Various elasticity coefficients are defined for the outcome variables and utility preferences attached to them. They indicate whether a variable possibly takes on large negative values. The coefficients can also be defined as sample statistics and, thus, computed from observed data. The approach admits various applications to practical problems of disaster risk management. The applications include estimations of the effectiveness and cost-efficiency of risk management, the specification of limits of acceptance of catastrophic risk for regulatory purposes, and safety and security systems design and dimensioning.

**Keywords** Catastrophic risk · Disaster risk · Extreme events · Non-expected utility · Risk management

**JEL Classification** D81 · Q54 · C18 · C44

## 1 Introduction

Disaster risk analysis recently developed into, and was firmly established as, a field of quantitative research (de Haan & Ferreira, 2006; Garrick, 2008; Grossi & Kunreuther, 2005; Michel,

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2018; Novak, 2012). The progress made draws considerably on probabilistic extreme value theory, which is concerned with the maxima and minima of sample values of real random variables and the probabilistic modelling of their frequency distributions. It admits mathematical representations of catastrophic risk in terms of strongly left-skewed probability distributions with heavy tails on the left, corresponding to the notion of catastrophic risk as low-probability, high-impact incidents (Pisarenko & Rodkin, 2010, 2014). In risk research, it serves as a primary source of probabilistic models of rare random events involving extreme loss or damage such as major earthquakes, aeroplane crashes or outbreaks of severe infectious diseases.

The present paper develops a conceptual framework for disaster risk analysis and management which is complementary to the available quantitative approaches. It dispenses with probabilistic extreme value theory and refers almost exclusively to relative frequency and simple probability distributions instead. Disaster risk assessment will be conceptualised in terms of a recently developed model of generalised expected (i. e., non-expected) utility preferences defined on the set of risky choices available to an agent. Within the model, the utility values of catastrophic risks prove highly unstable ("elastic") in the sense that they respond disproportionately to small perturbations of the decision outcomes and their finite-sample frequency distributions even if the samples taken are large and random. The formalism is shown to admit numerous applications to practical problems of the early indication, quantitative assessment and management of rare extreme events.

There are important theoretical and practical reasons for considering alternatives to the existing quantitative models of extreme events. First, probabilistic models of catastrophic risk must specifically accommodate the asymptotic tail behaviour of heavy-tailed probability distributions. They are thus exposed to systematic error. Methodically, this "model uncertainty" is cumbersome to assess and control, and may lead to serious underestimates of the losses incurred in disasters (for a recent, broad overview of the relevant literature in different application areas of tail modelling and analysis, see Visser & Petersen, 2012; Visser et al., 2014, 2015; Lam & Mottet, 2017; Huang and Lam 2019). These problems are exacerbated by the dearth of information on extreme events, which leaves models difficult to test against actual historical data or to validate by scenario-based computer experiments, notably so in case of non-repetitive occurrences ("one-shot risks") and extreme events without historical precedents. The latter case is typical, for example, of unprecedented weather extremes in response to rapid global warming or hitherto inconceivable, large-impact safety risks induced by technological innovations. Furthermore, probabilistic predictions of catastrophic events may be unreliable and require non-stationarity models of extremes as well as elaborate analytical and numerical techniques to inform the practice of disaster anticipation and prevention (Olsen et al., 1998; Renard et al., 2013; Serinaldi & Kilsby, 2015; Visser & Petersen, 2012; Visser et al., 2015). To avoid these problems, concepts of model robustness (insensitivity of results to model misspecifications or small perturbations of the model parameters) have been introduced into to quantitative disaster risk analysis (Ermoliev & Hordijk, 2006; Ermoliev et al., 2012; Guin, 2018). The present investigation develops and confirms a distinct approach within this framework.

Secondly, the probabilistic modelling of catastrophic risks is widely viewed as a basic requirement to improve risk management decisions (e. g., Garrick, 2008; Grossi & Kunreuther, 2005). However, the probabilistic modelling approaches currently taken in disaster risk analysis typically dispense with key concepts of optimal ("rational") risky choice such as risk preferences and their utility representations. More generally, major areas of quantitative risk and decision research do not make explicit reference to risk preferences at all, for instance, theories and applications of risk measures developed in financial and actuarial

mathematics (for analysis of this situation, see Denuit et al., 2006; Goovaerts et al. 2010; Drapeau & Kupper, 2013; Kou et al., 2013), and probabilistic risk and safety analysis (PRA, PSA) in the engineering sciences (Grechuk & Zabarankin, 2014; Johansen & Rausand, 2014; Rausand & Haugen, 2020). From the perspectives of the present investigation, at least two reasons can be given for this discrepancy. On the one hand, probabilistic risk measures and analyses are particularly suited for setting regulatory requirements of capital, solvency, public health and nuclear safety, and for assessing compliance with the regulations (see Bedford & Cooke, 2001, p. 259; Dhaene et al., 2006; Föllmer & Knispel, 2013; Kou et al., 2013), rather than improving management decisions in the sense of utility maximisation. On the other hand, concepts of expected and non-expected utility have thus far proved difficult to apply in quantitative assessments of catastrophic risks. Like the monetary risk measures, they tend to be very sensitive to distributional assumptions in these applications and, hence, need robustification (Grechuk & Zabarankin, 2014; Ikefuji et al. 2015; Ericson & Kruse, 2016). In the following, improvements of planning and decision making in disaster risk management will be based on the rigorous distinction between risk measurement (as defined exclusively in statistical and probabilistic terms) and risk assessment (as defined in terms of preference and human attitudes towards risk). As for the general significance of this distinction for decision science, see Cecconi et al. (2006), Drapeau and Kupper (2013), and Ericson and Kruse (2016).

The scope of the present analysis can be outlined in more technical terms as follows. We aim at assessing rare extreme events in systematic (rather than ad hoc), quantitative and consistent ways. The assessment methodology is designed to enhance disaster prevention planning, decision making and related risk management activities. To cope with the problems of model robustness and sensitivity to distributional assumptions, we restrict the analysis to simple probability functions and the random variables with finite ranges by which the functions are determined. Technically, in our approach risk measurement means sampling random variables. However, since we are mostly concerned with statistical quantities directly computed from the sample data, we need not explicitly estimate or model the underlying probability distributions. To assess measured risks, we further restrict the analysis to sample frequency distributions, their statistics and numerical risk preference values. Risk preferences will be represented using a generalised expected (non-expected) utility framework recently introduced in the literature (Geiger, 2008, 2020). It accounts for basic empirical properties of risky choice known from behavioural economics. In addition to its descriptive power, the approach offers a model of "pragmatic rationality": it satisfies familiar rationality principles of risky choice, with explicit reference being made to basic and frequently observed circumstantial constraints on decision making. They include the needs and demands (aspiration level) of the decision maker, his current economic status, health, exposure to natural hazards or technological risks (status quo), and time constraints on risk exposure (Kahneman & Tversky, 1979; Diecidue and van de Ven 2008; Geiger, 2008, 2020; Diecidue et al., 2015). The generalised expected utility approach taken has two prominent features on which the present analysis builds. First, it admits an explicit, algebraically simple expression for the certainty equivalent of a risk (Geiger, 2015). By the certainty equivalent ("intrinsic value", "preference value") of a risk with outcome variable $X$ and probability distribution $p$ we mean the unique existence[1] of a real number $c_p$ so that the decision maker is indifferent in preference between receiving the uncertain outcome $X$ or the sure amount $c_p$. Secondly, the certainty equivalent

[1] In the underlying non-expected utility approach, the existence of a certainty equivalent follows immediately from the result that the generalised utility function is uniquely defined everywhere on the real axis, smooth and strictly increasing. Smoothness of the utility function, in turn, is to be attributed essentially to an axiom of "transparent" first-order stochastic dominance preference (see Geiger 2008, 2020).

$c_p$, and, hence, the utility preference of $p$ turn out to be highly risk-sensitive in case $X$ takes on large negative values even with small probabilities. An index measuring the variability of highly risk-sensitive preferences of this kind will be defined. It is referred to as the *risk elasticity of the certainty equivalent* of $p$. It remains well-defined and will be shown to be finite, although possibly very large, provided that $c_p \neq 0$. Thus, if $q$ is the relative frequency distribution of a sequence of measurements of the random variable $X$ with unknown probability distribution $p$ and with the sample risk elasticity of $c_q$ being positive and large, the measurements indicate that $X$ may take on exceptionally large, negative values even if such extreme outcomes have not yet occurred. This is basically what we mean by early indication, or *ex ante* identification, of catastrophic risk.[2] The concept of indication implies that the measurements provide limited evidence for extreme events of a specific nature to happen, although with unknown probabilities and at uncertain times. It does not imply that the large risk elasticity of a measured certainty equivalent constitutes a necessary or sufficient criterion of a catastrophic incident to occur, nor does it attempt to estimate the probabilities of extreme events. However, it does provide useful information, and methodological tools and criteria for disaster risk management, especially for coping with problems of risk planning and decision making. The problems include estimation of the effectiveness of (degree of risk reduction achieved by), and cost-efficiency of (degree of risk reduction achieved per unit of money invested in), disaster risk-preventive measures. Other such problems are specifications of the limits of acceptance of catastrophic risk for regulatory purposes and the sizing of (technical, organisational, medical, etc.) safety and security measures.

The analysis proceeds as follows. The next section summarises the analytical framework of non-expected utility theory used in the approach. Brief explanations of the meaning and significance of the theory and its basic implications will be added where necessary. Section 3 extends the formalism to the quantitative assessment of catastrophic risk. In Sect. 4, the key indicators of catastrophic risk are introduced, and their validity and limits are discussed from methodological perspectives. Two application examples are presented in Sect. 5. Section 6 characterises various typical disaster risk management tasks which can be treated with the use of the disaster risk indicators developed. Section 7 closes with a brief discussion and an outlook on theoretical extensions and further management applications of the approach.

## 2 The underlying utility model: notation and summary

The non-expected utility model used in the present investigation is summarised to the extent required by the analysis in the subsequent sections. Similarly, the notation used in the approach needs some preparatory effort to give the certainty equivalent of disaster risk the simple algebraic representation which the risk assessment model exploits. For a more detailed account, axiomatic representation of the underlying principles of rational risky choice and realistic applications, see Geiger (2008, 2015, 2020), and Petzel et al. (2015).

By a risk we mean any real random variable $X$ (or, equivalently, the probability distribution it determines) measuring the quantitative consequences of an unintended, or "natural", event (safety risk) or intentional human action (security risk) being potentially harmful to individuals and groups. Without loss of generality, we concentrate on random variables with finite ranges within a given compact real interval $I$ with interior point 0. Let $P$ be the convex

[2] The term „early warning" might sound more familiar than "early indication". Early warning is often used to refer to an imminent threat, however. In contrast, the present conceptual framework specifically serves the requirements of long-term risk management strategies, which involve planning and preparatory action ahead of time.

set of simple probability distributions defined on $I$. Thus, every $X$ considered is distributed according to some $p, p \in P$. A person's attitude towards a given risk is assumed to be governed by this person's neutral reference point, or aspiration level $x_0$, $x_0 \in I$, status quo risk with probability distribution $s$, $s \in P$, and relative persistence $\omega$ of $p$ in the presence of $s$. "Relative persistence" means the overall probability of $T_p > T_s$, that is, $\omega = \Sigma_{t_p > t_s} d(t_p, t_s)$, where $T_p$ and $T_s$ are random variables measuring the uncertain risk resolution times of $p$ and $s$, respectively, and $d(t_p, t_s)$ is the joint probability that $T_p = t_p$ and $T_s = t_s$.[3] Thereby, $T_p$ and $T_s$, too, are supposed to vary in a finite discrete fashion over suitably chosen time intervals, "$\Sigma_{t_p > t_s}$" denoting summation over all pairs $t_p, t_s$ in the support of $d$ with $t_p > t_s$. In addition, $X$ and the real random vector $(T_p, T_s)$ are supposed to be independent. We also note that the account of non-expected utility underlying the present approach is based on the stochastic independence of each $p$ and $s$ of $d$. While $\omega$ generally varies with $p$ for given status quo $s$, $x_0$ and $s$ are exogenously given. To avoid trivial cases, $s$ is non-degenerate and involves at least some chance of gain $s(x) > 0$ for some $x > x_0$, $x \in I$, and some risk of loss $s(x') > 0$ for some $x' < x_0$, $x' \in I$. Without loss of generality, $x_0$ is normalised to $x_0 = 0$.

Status quo dependence of the preference order implies that the agent confronts the risk $s$ while ranking his preferences on $P$. It poses the problem of whether, once selected, $p$ is resolved prior to the resolution of $s$, or conversely, $s$ is resolved first and $p$ persists. How to estimate or compute the probability $\omega$ that $p$ persists while $s$ is resolved first, is a problem which is non-specific to the present approach, however. It can be treated within established methodological frameworks of statistical lifetime and failure data analysis. Here, it suffices to note that the present approach explicitly acknowledges that, together with status quo dependence, the time to resolution of risk, $T_p$, constrains the assessment of $p$ (see also Epstein & Kopylov, 2007; Eisenbach and Schmalz 2013). As such, $T_p$-dependence is not related to the problems of temporal risk and temporal preference, nor to the dynamics of choice (Geiger, 2020). It will, however, be employed below to model disaster return periods such as those of 100-year floods in hydrologic risk management.

Risks are represented with reference to their overall probabilities of gain and loss. Let $F_p$ be the cumulative distribution of $p$. Then, $\lambda_p = (1-F_p(0))/(1-p(0))$, $p(0) \neq 1$, and $1-\lambda_p$ are the overall probabilities of gains and losses, respectively, while $p$ can be written as a convex combination $p = p^+\lambda_p + p^-(1-\lambda_p)$ of $p^\pm$, where $p^+(x) = p(x)/\lambda_p$ and $p^-(x) \equiv 0$ for $x > 0$, and $p^-(x) = p(x)/(1-\lambda_p)$ and $p^+(x) \equiv 0$ for $x < 0$, if $0 < \lambda_p < 1$, $x \in I$. In addition, $p^+(0) = p^-(0) = p(0)$. If, on the other hand, $\lambda_p = 1$ or $\lambda_p = 0$, then $p^\pm = p$, respectively. The expected gain (loss) from $p$ is $\mu_p{}^\pm = \Sigma_{x \in S} x p^\pm(x)$, $S$ being the support of $p$, $S \subset I$, with the mean value $\mu_p = \mu_p{}^+\lambda_p + \mu_p{}^-(1-\lambda_p)$. There exists a generalised expected (i. e., non-expected) utility functional $U$: $P \to \mathbb{R}$ so that $U(p) \geq U(q)$ for every pair $p \in P$, $q \in P$, if and only if $p$ is preferred or indifferent to $q$, respectively. As such, it is uniquely determined up to monotonic, increasing transforms of the utility scale by three familiar axioms of rational preference (axioms of weak ordering, continuity and a weak version of first-order stochastic dominance preference) and one additional principle which postulates preferences on $P$ to depend on the status quo risk of the decision maker. Let $\hat{x}$ denote the degenerate probability distribution which gives the outcome $x$ with certainty. Then, for every $p$, $p \in P$, there exists a uniquely determined certainty equivalent $c_p$, $c_p \in I$, so that $U(p) = U(\hat{c}_p)$, and

$$U(p) \geq U(q) \Longleftrightarrow U\left(\hat{c}_q\right) \geq U\left(\hat{c}_q\right) \Longleftrightarrow c_p \geq c_q, \qquad p \in P, q \in P \tag{1}$$

[3] Otherwise, $\omega$ is known as the reliability parameter of bivariate lifetime models in failure and reliability analysis. For reliability models with bivariate exponential lifetime distributions for instance, see Downton (1970), Krishnamoorthy et al. (2007), and Geiger (2020). Here, the term "relative persistence" would seem more appropriate, however, since $\omega$ gives the overall probability of $s$ being resolved first while $p$ persists.

One also has $c_p = \mu_p$ ("risk neutrality", i. e., $p$ is exactly worth its expected value) or else $c_p \lesseqgtr \mu_p$, meaning risk aversion (proneness), that is, $p$ is worth less (more) than its expected value. Furthermore, $c_s = 0$, corresponding to the notion that choosing $s$ amounts to remaining in the status quo and, hence, gaining or losing 0 with certainty. Finally, for every $p \in P$ with $0 < \lambda_p < 1$, there uniquely exist a real number ${\lambda_p}^0$, $0 < {\lambda_p}^0 < 1$, and a convex combination $p^0 = p^+{\lambda_p}^0 + p^-(1-{\lambda_p}^0)$ so that $p^0$ and $s$ are indifferent in preference, that is, ${c_p}^0 = c_s = 0$. The overall probability of gain, ${\lambda_p}^0$, is determined by

$$z_s = \frac{\mu_s^+\lambda_s + \mu_s^-(1-\lambda_s)}{\left(\mu_s^+ - \mu_s^-\right)\sqrt{\lambda_s(1-\lambda_s)}} = \frac{\mu_p^+\lambda_p^0 + \mu_p^-(1-\lambda_p^0)}{(\mu_p^+ - \mu_p^-)\sqrt{\lambda_p^0(1-\lambda_p^0)}} \tag{2}$$

(Geiger, 2008 Sec. 5). Denoting the certainty equivalents of $p^\pm$ and $p^0$ by ${c_p}^\pm$ and ${c_p}^0$, respectively, one altogether has

$$\mu_p^- = c_p^- < c_p^0 = c_s = 0 < c_p^+ = \mu_p^+ \tag{3}$$

From these definitions and results, one finds,

$$c_p = \mu_p \Phi(\lambda_p, \lambda_p^0) \tag{4}$$

where $\Phi$ gives the deviation of $c_p$ from the risk neutral case $c_p = \mu_p$,

$$\Phi(\lambda_p, \lambda_p^0) = \frac{\mu_p^+(1-\omega)(\lambda_p - \lambda_p^0)}{\mu_p(\omega(1-\lambda_p) + (1-\omega)(1-\lambda_p^0))}, \lambda_p \geq \lambda_p^0, \tag{5}$$

$$\Phi(\lambda_p, \lambda_p^0) = \frac{\mu_p^-(1-\omega)(\lambda_p^0 - \lambda_p)}{\mu_p(\omega\lambda_p + (1-\omega)\lambda_p^0)}, \lambda_p^0 \geq \lambda_p \tag{6}$$

(Geiger, 2015). The initial value of $x_0$ must be added on the right-hand side of Eq. (4) in case the normalisation $x_0 = 0$ is cancelled. It is also important to note that in comparisons $c_p > c_q$, different parameter values $\omega_p$ and $\omega_q$ must be inserted for $\omega$ in Eqs. (5) and (6) if $p$ and $q$ are different by relative persistence. This $\omega$-dependence of comparative risk assessment typically arises in reliability management applications where safety components design and dimensioning serve to decrease the failure rates of the systems or processes to be managed. Demonstrations of this effect of failure rate management will be given below.

According to Eqs. (4–6), $c_p$ is simple to compute from $p$ and the parameters $x_0$, $s$ and $\omega$. Together with Condition (1), the present formalism should therefore be of considerable practical use to assess risks systematically and quantitatively. Some more detailed results can be resumed as follows. The limits of $\Phi(\lambda_p, {\lambda_p}^0)$ for $\lambda_p \to 0$ and $\lambda_p \to 1$ exist. Hence, the representation (4) holds for every $p, p \in P$. One has $\Phi(0, {\lambda_p}^0) = \Phi(1, {\lambda_p}^0) = 1$, corresponding to "pure chance" $p = p^+$ and "pure risk" $p = p^-$. The latter is also known as "downside risk". In these special cases, risk neutrality $c_p = \mu_p = {\mu_p}^\pm$ obtains. Decision makers are risk averse (prone) towards $p^0$ if ${\mu_p}^0 > 0$ (${\mu_p}^0 < 0$), and similarly for the status quo $s$ with $\mu_s > 0$ ($\mu_s < 0$). More generally, risk aversion (proneness) holds if $\Phi(\lambda_p, {\lambda_p}^0) < 1$ ($\Phi(\lambda_p, {\lambda_p}^0) > 1$), whereas $\Phi({\lambda_p}^0, {\lambda_p}^0) = {c_p}^0 = 0$ even in cases in which ${\mu_p}^0$ does not vanish. If $p$ persists while $s$ resolves first with certainty ($\omega = 1$), $c_p = 0$ holds as well. In fact, if $p$ is selected in the presence of $s$, but $s$ is resolved first with certainty, $p$ survives as the new status quo. For later reference, we note this result as

$$c_p = c_p^0 = c_s = 0, \omega = 1, \rho \in P \tag{7}$$

Conversely, if $\omega = 0$, $p$ resolves first with certainty while $s$ persists. If, in addition, $\mu_s = 0$, then the entire preference order on $P$ is risk neutral, that is, $\Phi(\lambda_p, {\lambda_p}^0) = 1$. One easily

verifies this result, recalling that $\Phi(1, \lambda_p{}^0) = \Phi(0, \lambda_p{}^0) = 1$ and considering that $\mu_p{}^0 = \mu_s = 0$ if $0 < \lambda_p < 1$, by Eq. (2) (derive $\lambda_p{}^0 = -\mu_p{}^-/(\mu_p{}^+ - \mu_p{}^-)$ from $\mu_p{}^0 = 0$ and replace $\lambda_p{}^0$ in Eqs. (5) and (6) accordingly). Finally, a risk $p$ satisfying $c_p \geq c_s = 0$ is viewed as acceptable in the sense of being (weakly) preferred to the status quo. The exceptions are disaster risks. They must meet an additional, more restrictive requirement even if they satisfy $c_p \geq 0$. This case will be considered separately below.

Figure 1a depicts $c_p$ as a function of $\mu_p$ for a risk averse status quo with $\mu_s > c_s = 0$ and, because of Eq. (2), $\mu_p{}^0 > c_p{}^0 = 0$. It illustrates that there exist risks with positive $\mu_p$ for which $c_p$ is negative. Conversely, if his status quo is unfavourable ($\mu_s < 0$ and, hence, $\mu_p{}^0 < 0$), the decision maker may feel forced to "gamble for resurrection" and turn risk prone ($c_p > \mu_p$). He thus assigns a positive $c_p$ to $p$ even though $\mu_p$ is negative (Fig. 1b, hatched area). This type of preference ranking must be expected to obtain in what may be called decision making under risk and despair, corresponding to a "desperate" status quo with an almost sure, large negative expected outcome ($1-\lambda_s >> \lambda_s$, $\mu_s < 0$), but nonetheless $c_s = 0$. Finally, Fig. 1a shows the risk premium $\pi_p = \mu_p - c_p$ as the amount by which the expected outcome of the risk $p$ must exceed the risk-free amount $c_p$ in order for the decision maker to be indifferent between $p$ and $\hat{c}_p$. The risk premium is positive for risk aversion and negative for risk proneness.

## 3 Catastrophic risk

Like gains and losses in general, extreme negative outcomes of random events are defined with reference to the aspiration level. We first exclude the trivial case $p = p^-$ in which $p^+ = \hat{0}$. In this instance of downside risk, Eq. (3) entails $c_p = c_p{}^- < c_s = 0$, meaning that $p$ is unacceptable: the decision maker prefers to remain in the status quo rather than choose $p$, irrespectively of whether resolution of the risk $p$ may entail a disaster or not. Now assume that $\mu_p{}^+ > 0$. The risk $p$ is defined to involve extreme losses if it is heavily skewed to the left about $x_0$, that is, $\mu_p{}^-$ is far below what the decision maker expects or needs to gain, at a minimum of $x_0$, from choosing $p$,

$$-\mu_p^- >> \mu_p^+ \tag{8}$$

The dependence of Condition (8) on $x_0$ can be made explicit for arbitrary $x_0$ prior to the normalisation $x_0 = 0$ roughly as follows: for $\mu_p{}^+ > x_0 > 0$, Inequality (8) gives $-\mu_p{}^- + x_0 >> \mu_p{}^+ + x_0$ so that $-\mu_p{}^- + x_0 >> x_0$ and $-\mu_p{}^- >> x_0$ also hold, and similarly $\mu_p{}^- << x_0$ for $x_0 < 0$.

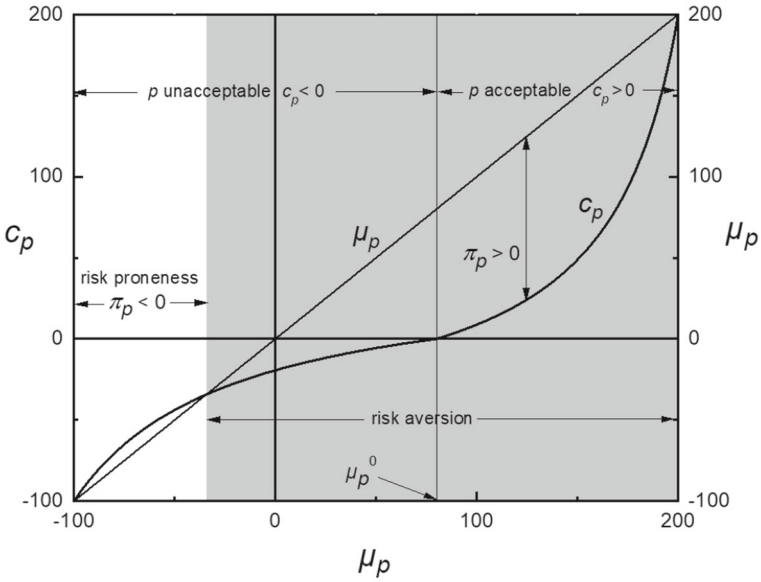
Returning to the normalisation $x_0 = 0$, we distinguish six types of comparison of $\mu_p{}^0$ and $\mu_p$ of the form $\mu_p \leq \mu_p{}^0 \leq 0 < \mu_p{}^+$, which are compatible with Inequality (8). Three of them involve the inequality $\mu_p \leq \mu_p{}^0$ and, hence, $c_p \leq c_s = 0$, as can be easily seen from Fig. 1. So $s$ is weakly preferred to $p$ in these cases, which can be ignored. Two other comparisons,

$$\mu_p^0 \leq 0 < \mu_p \leq \mu_p^+ \quad \text{and} \quad \mu_p^0 \leq \mu_p \leq 0 < \mu_p^+ \tag{9}$$

imply $c_p \geq c_s = 0$, as is obvious from Fig. 1b, but these can be disregarded as well (see below). The following analysis is concerned with the last and only relevant case,

$$0 < \mu_p^0 \leq \mu_p \leq \mu_p^+ \tag{10}$$

(a)




(b)

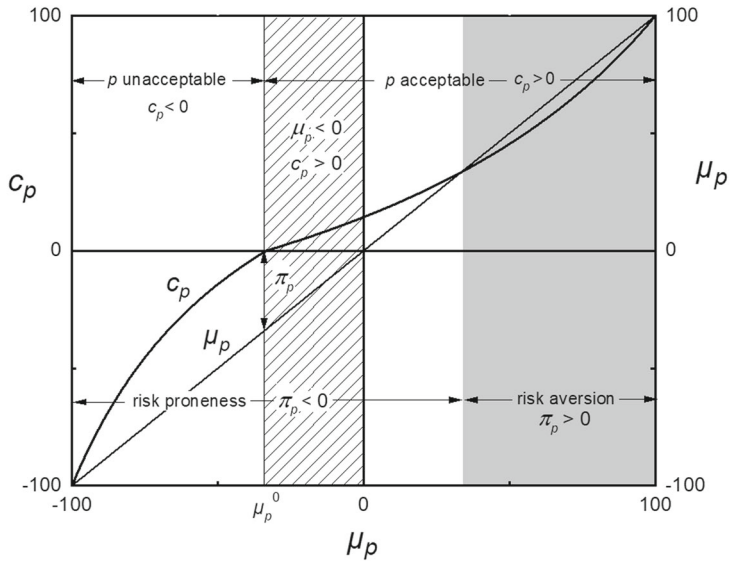



**Fig. 1** Certainty Equivalent as a Function of Mean Value of $p$. Various areas of risk attitude expressed by risk premium $\pi_p = \mu_p - c_p \gtrless 0$ for different values of the governing parameters. The shaded areas signify risk aversion $\pi_p > 0$. **a** Risk averse status quo $s$ with $\mu_s > 0$ and $\mu_p{}^0 > 0$. **b** Unfavourable status quo risk $s$ with $\mu_s < 0$ and $\mu_p{}^0 < 0$. The region in which risk proneness prevails includes the hatched area with positive certainty equivalent for negative expected outcome of $p$

Together with Inequality (8), it requires that, for $c_s = 0 \leq c_p$ to hold, $p$ must satisfy$\mu_p{}^0 \leq \mu_p$ and, hence, $\lambda_p{}^0 \leq \lambda_p$, while $\lambda_p{}^0$ must already be large and close to unity,

$$1 \gtrsim \lambda_p^0 \geq \frac{-\mu_p^-}{\mu_p^+ - \mu_p^-},$$

Hence, losses and, especially, extreme losses from $p$ are rare,

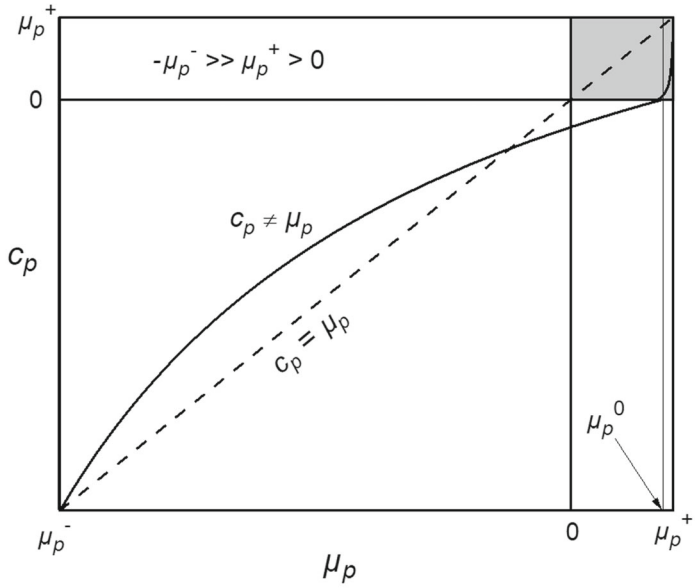
$$1-\lambda_p << \lambda_p \tag{11}$$

so one also has $\mu_p \lesssim \mu_p{}^+$.Inequality (11) explains why the special cases (9) can be excluded from the present analysis. In fact, the inequalities in (9) apply to values of $\mu_p$ markedly smaller than $\mu_p{}^+$. These values violate Inequality (11) since large losses are not necessarily rare in such cases. Real-time forecasting and early warning of an imminent threat, rather than disaster early indication in the sense of the present analysis, would seem more appropriate management approaches to situations in which Inequalities (9) apply. A similar situation obtains in what has been referred to above as decision making under risk and despair, in which the status quo risk is already highly unfavourable. If so, even a risk $p$ satisfying Condition (8) may have positive $c_p$ if it violates (11). This example of $c_p > 0$ for $\mu_p < 0$ is similar to the case shown in Fig. 1b, in which the inequalities on the right-hand side of (9) hold. However, disaster risk anticipation is trivial if (8) holds while (11) is violated.

Informally, Inequalities (10) and (11) provide a firm prospect of a positive overall gain in view of a very unlikely, albeit massive, overall loss. At least, this is what the positive certainty equivalent of $p$ suggests for $\lambda_p \geq \lambda_p{}^0$ according to Inequality (10). It means that $p$ is an improvement over the status quo despite the large losses to which it may give rise. It is this prospect which may induce a "false sense of safety or security" in decision makers, exacerbating the need for early indication of catastrophic risk in practice. Meanwhile, Inequality (11) raises the question of whether $1-\lambda_p << \lambda_p$ demands too much by referring to the overall probability of loss rather than, more specifically, to the probability of extreme loss. To clarify this point, assume that Inequality (11) is violated while (8) holds. From Eq. (2) and Condition (10), one then has $1-\lambda_p{}^0 << \lambda_p{}^0$ and $\mu_p \leq \mu_p{}^0$ from the violation of Inequality (11). The latter result does not only contradict Condition (10) for $\mu_p > \mu_p{}^0$, but also leads back to the trivial case $c_p \leq 0$ discarded above. Finally, but most importantly, even the risk of an extreme negative outcome may be very small if only $\lambda_p$ is sufficiently close to unity. How sufficient, then, is sufficient enough in terms of disaster risk reduction as a risk management task? This question calls for criteria more refined than Conditions (8) and (11) to make the present concept of disaster risk fully applicable. They will be developed in the next section.

Continuing restriction to the non-trivial case $\mu_p{}^+ > 0$, we further simplify the formalism by scaling outcomes $x$ to $x/\mu_p{}^+$, thus normalising $\mu_p{}^+$ to 1, and rewrite $\mu_p/\mu_p{}^+$ as $\mu_p$, $\mu_p{}^-/\mu_p{}^+$ as $\mu_p{}^-$, etc. Condition (8) now gives $-\mu_p{}^- >> 1$. By Eqs. (4–6) and the equivalences in (1), $U(p)$ and $c_p$ are implicit, positive homogeneous functions of $x$ (cf. Geiger, 2015). The transformation $x \rightarrow x/\mu_p{}^+$ thus leaves comparisons of the form (1) invariant if $\mu_p{}^+ = \mu_q{}^+$ or $c_q = 0$ (esp., in case $q = s$ or $q = p^0$). Otherwise, $p$ and $q$ will be tacitly assumed in (i. e., transformed back into) their natural outcome scaling when to be compared in preference. Figure 2 shows the typical shape of the $c_p$-curve as a function of $\mu_p$ for a disaster risk $p$ satisfying Conditions (8) and (11), with a large negative $c_p$ in a wide range of $\lambda_p$- and $\mu_p$-values. A positive $c_p$ exists only for $\lambda_p{}^0$ and for $\lambda_p$-values close to 1, or, equivalently, in a narrow interval$\mu_p{}^0 < \mu_p \leq 1$. The steep decline of the $c_p$-curve for decreasing $\mu_p$ between 1 and $\mu_p{}^0$ means that, as a catastrophic risk, $p$ is unstable to small random perturbations of $\mu_p$ (or, equivalently, $\mu_p{}^-$ or $\lambda_p$) and may strongly diminish in intrinsic value even under minor

(a)




(b)

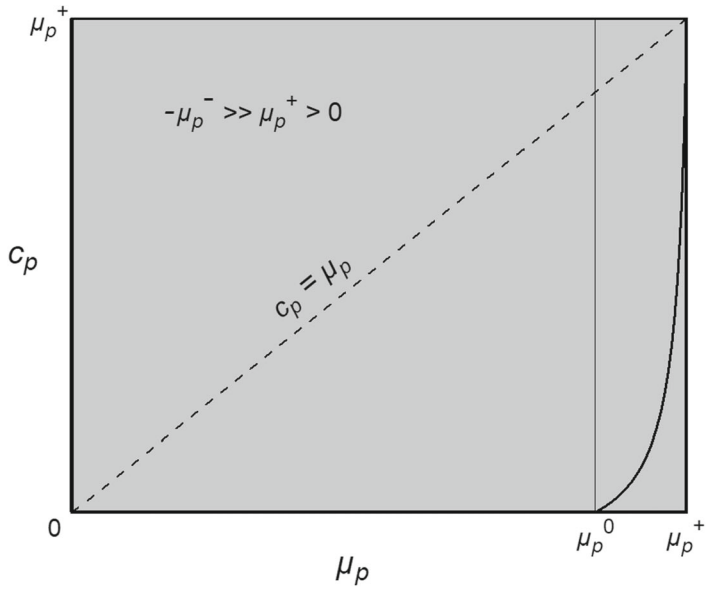



**Fig. 2** Certainty Equivalent of Catastrophic Risk $p$. Typical shape of the $c_p$-curve as a function of $\mu_p$ for disaster risk $p$ (schematic representation). The shaded areas in (a) and (b) are the same. **a** For very low probability of large loss, $c_p$ is positive (shaded area with ${\mu_p}^0 < \mu_p \leq 1$). **b** Steep decline of the $c_p$-curve for decreasing $\mu_p$ between ${\mu_p}^+ = 1$ and ${\mu_p}^0$. Decreasing $\mu_p$ means increasing probability of a large overall loss

changes of $\mu_p$. Note that this characteristic feature of disaster risk arises already in finite discrete representations and utility assessments of risk. In contrast to extreme value models, asymptotic behaviour of probability distributions is not involved here.

## 4 Indicators of catastrophic risk

### 4.1 Indication of risk

An appropriate formal representation of risk indication is conditioning one random variable on another in the usual sense, given simple probability distributions. Let $f$ be the probability function of the jointly distributed random variables $X$ and $Y$, with the marginal distributions $f_X = p$ and $f_Y = q$. The event that $X = x$ conditional on $Y = y$, $q(y) \neq 0$, has the probability $f_{X|Y}(x|y)$ in the usual notation. The event that $Y = y$ is said to *indicate* (i. e., *provide evidence for*) the event that $X = x$ if $f_{X|Y}(x|y) > f_X(x)$ and, similarly, to *provide counterevidence to* the event that $X = x$ if $f_{X|Y}(x|y) < f_X(x)$, and to provide no indication that $X = x$ in case of independence of $X = x$ and $Y = y$, that is, $f_{X|Y}(x|y) = f_X(x)$. This terminology also draws a distinction between strong and weak indication in terms of, respectively, a large or small gain in the probability that $X = x$ if, in addition, $Y = y$ is given. It also covers the implication (exclusion) of $X = x$ by $Y = y$ as the strongest possible instances of indication: $Y = y$ is equally probable as the joint event that $Y = y$ and $X = x$ ($X \neq x$), that is, $f_{X|Y}(x|y) = 1$ ($f_{X|Y}(x|y) = 0$).

### 4.2 Risk elasticity of the certainty equivalent

A given relative increase in risk $-\mathrm{d}\mu_p/\mu_p$, as expressed in terms of decreasing $\mu_p$, may not only lead to a sudden large disproportionate reduction of the preference value of $p$, but also to relative changes $-\mathrm{d}c_p/c_p$. The latter may themselves vary considerably with $\mu_p$ and also with the parameters ${\mu_p}^0$, $\omega$, etc. This high sensitivity of risk assessment with which the decision maker responds to potential massive losses is another characteristic feature of catastrophic risk in the present risk assessment model. It motivates the following definition of the *risk elasticity of the certainty equivalent*,

$$\eta_p = \frac{\mu_p}{c_p} \cdot \frac{\mathrm{d}c_p}{\mathrm{d}\mu_p}, c_p \neq 0 \tag{12}$$

The derivation of $\eta_p$ as a function of $\mu_p$ is straightforward, yet lengthy and tedious. The essential steps are outlined in Appendix A. It suffices to note the result for the relevant interval ${\mu_p}^0 < \mu_p \leq 1$,

$$\eta_p = A_p \frac{\mu_p}{\mu_p - \mu_p^0} \cdot \frac{1-\mu_p^0}{1-\mu_p^0-\omega(\mu_p-\mu_p^0)} \tag{13}$$

$$A_p = \frac{2(\mu_p^0-\mu_p^-)}{\mu_p^0-\mu_p^-(2-\mu_p^0)} \tag{14}$$

The parameter $A_p$ does not explicitly depend on $\mu_p$, but strictly increases as a function of${\mu_p}^0$ between $A_p = 1$ and $A_p = 2$ for${\mu_p}^0 = 0$ and ${\mu_p}^0 = 1$, respectively. However, since we always have ${\mu_p}^0 \lesssim 1 << -{\mu_p}^-$, $A_p$ can be assumed constant and roughly equal to 2.

Figure 3 shows $\eta_p$ as a function of $\mu_p$ for various values of ${\mu_p}^0$, ${\mu_p}^-$ and $\omega$. If $\mu_s = \omega =$

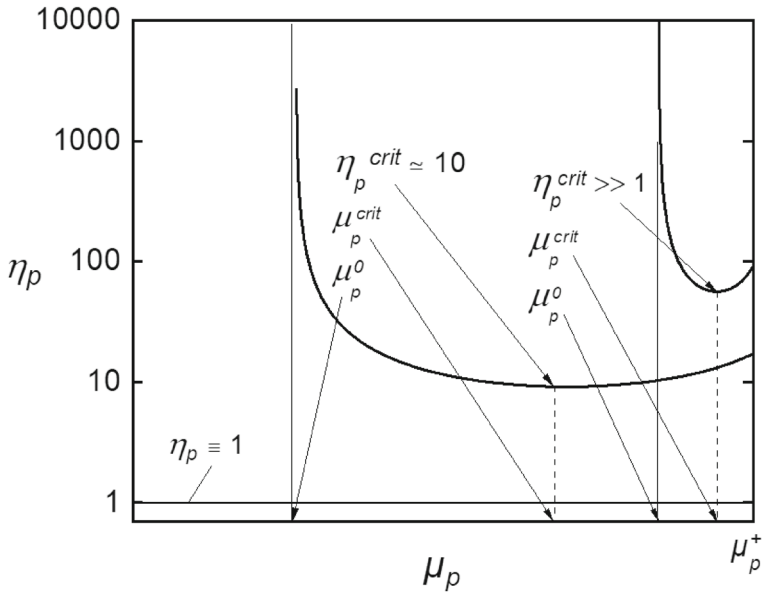



**Fig. 3** High Elasticity $\eta_p$ of Certainty Equivalent of Catastrophic Risk $p$. Elasticity coefficient $\eta_p >> 1$ as a function of $\mu_p$ for various values of ${\mu_p}^0$, ${\mu_p}^-$ and $\omega$, with minimum ${\eta_p}^{crit} \leq \eta_p$

0, one has ${\mu_p}^0 = 0$ for every $p \in P$ with $0 < \lambda_p < 1$, according to Eq. (2). Then, risk neutrality $c_p = \mu_p$ holds so that $\eta_p = 1$, consistently with Eqs. (12–14). Now, let ${\mu_p}^0 > 0$. Since in Eq. (2) $z_s = constant$ for given $s$, one finds that

$$-z_s^2 \mu_p^- \simeq \frac{\left(\mu_p^0\right)^2}{1-\mu_p^0}$$

so ${\mu_p}^0$ approaches 1 as${\mu_p}^-$ decreases and goes to minus infinity. It follows that, for positive values ${\mu_p}^0$ well below 1, $p$ with ${\mu_p}^0 < \mu_p < 1$ is an intermediate risk satisfying $1 \lesssim \eta_p \simeq 10$, whereas

$$\eta_p >> 1, \mu_p^0 < \mu_p < 1 \tag{15}$$

under the conditions (8) and (11) of catastrophic risk $p$ with $c_p > 0$.

Although Inequality (15) follows from the conditions of rare extreme events stated in Sect. 3, characterising $p$ as a disaster risk solely in terms of a highly risk-elastic coefficient $c_p$ can be misleading. In fact, Condition (15) holds for every $p$ with $\mu_p$ close to ${\mu_p}^0$ since by definition $\eta_p \to \infty$ for $c_p \to {c_p}^0 = 0$. Similarly, in case of large $\omega$, $\omega \lesssim 1$, one has $\eta_p \to \mathrm{d}c_p/\mathrm{d}\mu_p >> 1$ for $c_p \to \mu_p \to 1$. To avoid these distracting boundary effects, we introduce the minimum ${\eta_p}^{crit}$ of the $\eta_p$-curve to signify a very large risk elasticity of $c_p$ in terms of a minimax criterion,

$$\frac{\partial \eta_p}{\partial \mu_p} = 0 \tag{16}$$

Equation (16) is uniquely satisfied for

$$\begin{aligned}\mu_p = \mu_p^{crit} &= \mu_p^0\left[1+\frac{1-\mu_p^0}{\omega\mu_p^0}\right]^{1/2}, 0 < \omega \le 1,\\ &= \mu_p^0(1+B_p)^{1/2},\end{aligned} \tag{17}$$

where $B_p = (1-\mu_p{}^0)(\omega\mu_p{}^0)^{-1}$. Equation (16) is intended to determine $\mu_p{}^{crit}$ and $\eta_p{}^{crit}$ for given $\mu_p{}^0$, that is, for fixed $\mu_p{}^-$, but variable $\lambda_p$. While $\eta_p$ is an explicit function of$\mu_p$ and $\mu_p{}^0$, $\mu_p{}^0$ also depends implicitly on $\mu_p$ (or $\mu_p{}^-$, for that matter): $\lambda_p{}^0$ together with $\mu_p{}^0$ are determined by $\mu_p{}^-$ according to Eq. (2) and $\mu_p{}^0 = \lambda_p{}^0 + \mu_p{}^-(1-\lambda_p{}^0)$, respectively. Hence the partial derivative (16). Inserting the result (17) into Eq. (13) gives

$$\eta_p^{crit} = \frac{A_p B_p}{2+B_p-2\left(1+B_p\right)^{1/2}} \tag{18}$$

while Condition (15) goes over into

$$\eta_p \ge \eta_p^{crit} \simeq 4A_p B_p^{-1} = \frac{4A_p\omega\mu_p^0}{1-\mu_p^0} >> 1 \tag{19}$$

for small $B_p \gtrsim 0$ (i. e., $\mu_p{}^0 \lesssim 1$). Generally, the strong inequality in Condition (19) may be reasonably understood to mean $\eta_p{}^{crit} \ge 100$. In practice, values of $\eta_p{}^{crit}$ smaller than 100 may arguably be viewed to indicate a growing risk of large losses as well. The latter may be expected to occur for $\eta_p{}^{crit} \ge 10$, by order of magnitude.

Equations (13–19) define $\eta_p$, $\mu_p{}^{crit}$ and $\eta_p{}^{crit}$ as the key indicators of disaster risk. Let $M$ and $H$ be random variables so that $M = 1$ and $M = \mu_p{}^-$ with probabilities $\lambda_p$ and $1-\lambda_p$, respectively, and, if $\mu_p > \mu_p{}^0$, $H$ with probability 1 for $H = \eta_p$, and 0 for $H \ne \eta_p$, $\eta_p \ge 1$. Using the notation adopted above, one has $1-\lambda_p = h_{M|H}(\mu_p{}^-|\eta_p)$ as the conditional probability with which the expected loss $\mu_p{}^-$ is obtained, given $\eta_p$. Recall that the preference order on $P$ is risk neutral if $\mu_p{}^0 = \omega = 0, p \in P$. In this case, Eqs. (13) and (14) entail $\eta_p = 1$ so that $h_{M|H}(\mu_p{}^-|1) = h_M(\mu_p{}^-)$, which means that $1-\lambda_p = h_M(\mu_p{}^-)$ is trivially independent of the event that $\eta_p = 1$. Nevertheless, $M$ and $H$ are dependent because of

$$h_{M|H}\left(\mu_p^-|\eta_p\right) \gtrless h_{M|H}\left(\mu_p^-|1\right) = h_M\left(\mu_p^-\right), \eta_p > 1 \tag{20}$$

Inequality (20) follows from the result that $h_{M|H}(\mu_p{}^-|\eta_p)$ is a strictly monotonic function of $\eta_p$,

$$\frac{\partial}{\partial\eta_p}h_{M|H}\left(\mu_p^-|\eta_p\right) = -\frac{\partial\lambda_p}{\partial\eta_p} = \frac{-1}{1-\mu_p^-}\left[\frac{\partial\eta_p}{\partial\mu_p}\right]^{-1} \lesseqgtr 0 \tag{21}$$

considering that $\mu_p = (1-\lambda_p)(\mu_p{}^- - 1) + 1$ and $\partial\eta_p/\partial\mu_p \gtrless 0$ for $\mu_p > \mu_p{}^{crit}$ and $\mu_p{}^0 < \mu_p < \mu_p{}^{crit}$, respectively (see Fig. 3).

In (20), the overall probability of $M = \mu_p{}^-$ decreases or increases, conditional on $H = \eta_p > 1$ and depending on whether $\mu_p \gtrless \mu_p{}^{crit}$ in (21). The boundary between negative and positive changes of probability of loss demarcates extreme events which are sufficiently improbable to tolerate the risk (in the sense that people fail to call for management responses to risk exposure) and those which are not. To see this, let $p$ satisfy Conditions (8) and (11), but also $c_p > 0$. Assume that $p$ is subject to small random perturbations of $\lambda_p$ which will result

in small changes $\Delta\mu_p$ and $\Delta c_p$. Estimating the average $\overline{\Delta c_p}$ of a large random sample of perturbations $\Delta c_p$, we show in Appendix A that

$$\overline{\Delta c_p} \simeq \frac{c_p}{\mu_p}(\Delta\mu_p)^2 \cdot \frac{\partial\eta_p}{\partial\mu_p} \tag{22}$$

Equation (22) states that when risks are subject to frequent, small random perturbations of the underlying probabilities—as is arguably always the case in real applications—the variations increase or decrease the certainty equivalent on the average, corresponding to $\partial\eta_p/\partial\mu_p > 0$ for $\mu_p > {\mu_p}^{crit}$ or $\partial\eta_p/\partial\mu_p < 0$ for $\mu_p < {\mu_p}^{crit}$. In other words, for probability values which are sufficiently large ($\lambda_p > {\lambda_p}^{crit}$), $p$ is robust if exposed to disturbances which are random and limited in size. The situation is a markedly different, unstable one if $\lambda_p < {\lambda_p}^{crit}$. We refer to it as acute catastrophic risk. When perturbed, it will likely deteriorate in safety or security even if initially $c_p$ was positive and if the perturbations are small. Moreover, if $p$ is recurrent and resolved repetitively, the situation in which the decision maker finds himself will "wear out" over time. This means that $c_p$ decreases continually and finally takes on negative values, which makes the disaster risk entirely conspicuous. For constant $\Delta\mu_p$, the relative deterioration ${c_p}^{-1}\overline{\Delta c_p} < 0$ will even proceed at an ever faster pace since $\eta_p \to \infty$ and $\partial\eta_p/\partial\mu_p \to -\infty$ as $c_p \to 0$.

Another important problem is how the disaster risk indicators respond to variations in the expected damage ${\mu_p}^-$ or, more generally still, how they depend on the total differential $\mathrm{d}\mu_p$. To solve this problem, the partial derivative (16) must be replaced by $\mathrm{d}\eta_p/\mathrm{d}\mu_p$. Besides $\partial\eta_p/\partial\mu_p$, the total derivative includes the term $(\mathrm{d}{\mu_p}^0/\mathrm{d}\mu_p)\partial\eta_p/{\mu_p}^0$. Neither $\mathrm{d}{\mu_p}^0/\mathrm{d}\mu_p$ nor $\partial\eta_p/{\mu_p}^0$ identically vanishes since $\eta_p$ depends explicitly on $\mu_p$ and ${\mu_p}^0$, but ${\mu_p}^0$ also on $\mu_p$ in an implicit, highly non-linear fashion, by Eq. (2). As a consequence, $\mathrm{d}\eta_p/\mathrm{d}\mu_p$ gives rise to some unwieldy algebra which leaves the explicit solution of $\mathrm{d}\eta_p/\mathrm{d}\mu_p = 0$ practically useless. In view of this situation, we proceed as follows. In a first step, we restrict the analysis to the indicator dependence on $\mu_p$, with ${\mu_p}^-$ and ${\mu_p}^0$ being held constant. Then, the impact of variations of ${\mu_p}^-$ on risk parameters is treated separately by considering various different $p$, $q$,… with distinct values ${\mu_p}^-$, ${\mu_q}^-$,… and ${\mu_p}^0$, ${\mu_q}^0$,… respectively. Finally, the distinct results will be compared. For a demonstration of this procedure in detail, see the application example in Sub Sect. 5.1.

A final point is the dependence of the disaster risk indicators on the exogenous parameters $x_0$, $s$ and $\omega$. We address this problem with reference to $\omega$ since the relative duration time of risk is critical for indicating whether a presumptive extreme event is acute. Figure 4 shows ${\mu_p}^{crit}$ as a function of $\omega$ and ${\mu_p}^0$, according to Eq. (17). If $\omega$ increases, ${\mu_p}^{crit}$ decreases along the curves ${\mu_p}^0 = constant$. The increase in $\omega$ means that smaller values of ${\mu_p}^{crit}$ admit risks $p$ to be effectively reduced and leave the latter more robust in the sense that they satisfy $\mu_p > {\mu_p}^{crit}$. In reliability engineering and management, strategies to decrease the failure rates of systems and processes (i. e., to increase $\omega$) are devised to achieve just this. Conversely, for constant $\omega$, ${\mu_p}^{crit}$ increases with ${\mu_p}^0$. Catastrophic risks $p$ must then yield values $\lambda_p$ and $\mu_p$ closer to unity for robustness to obtain because $\mu_p > {\mu_p}^{crit}$ is required while ${\mu_p}^{crit}$ is already large. Moreover, even a disaster risk $p$ with ${\mu_p}^0 \gtrsim 0.9$ and very small overall probability of loss,

$$1 - \lambda_p \lesssim \frac{1-\mu_p^0}{1-\mu_p^-} \lesssim 10^{-3}, \mu_p^- \lesssim -100$$

must be deemed acute if it is likely to be resolved prematurely (prior to the resolution of $s$), because ${\mu_p}^{crit} = 1$ for $\omega \leq 0.5$. An effective management strategy then is to defer disaster,

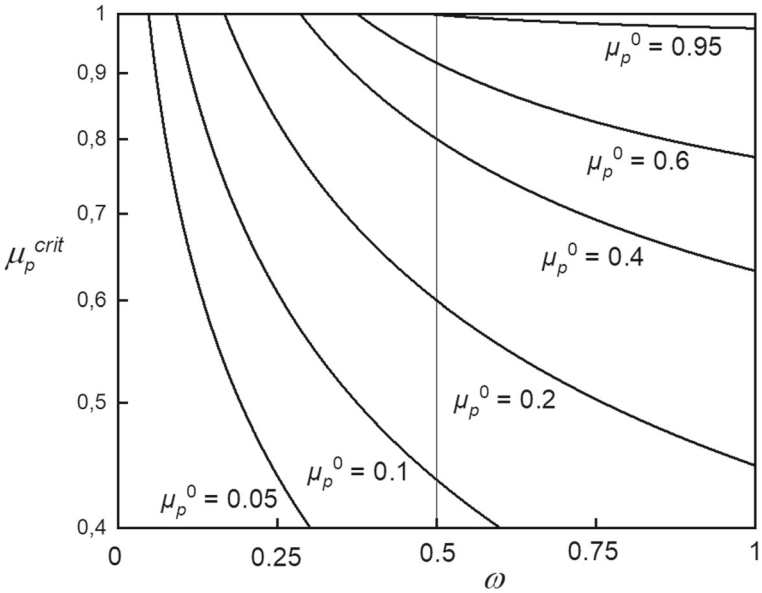



**Fig. 4** Critical Value of Expected Outcome $\mu_p{}^{crit}$ of Disaster Risk $p$ as a Function of $\omega$ and $\mu_p{}^0$. The parameter $\mu_p{}^{crit}$ separates the regime of robustness from that of instability (of the certainty equivalent) of catastrophic risk. The boundary between the two regimes varies with $\omega$ and $\mu_p{}^0$

or "buy time", by taking measures to increase $\omega$ beyond 0.5, if any. However, this procedure may not suffice if the reduction of $\mu_p{}^{crit}$ achieved is not large enough to decrease $\mu_p{}^{crit}$ below $\mu_p$. Additional risk management decisions to increase $\mu_p$ will then have to be taken, especially to reduce the overall probability or the expected amount of damage, or both.

### 4.3 Status quo risk: disaster deferred?

Risk analysis is often concerned not so much with decisions ahead, but with the uncertainties decision makers currently confront. The question may arise whether an extant risk $s$ must be viewed as a "disaster deferred" (Stein, 2010) rather than a more moderate risk, as is frequently encountered in everyday life. The conceptual framework of the preceding sections might seem unsuitable for treating this question since $c_s$ vanishes by definition so that $\eta_p$ is not defined for $p = s$ (Eq. 12). But the above formalism can be modified to indicate severe dangers inherent in the status quo, too. To this purpose, we conceive of $s$ as a risk selected by a person, or imposed on him or her, in some previously existing risky status quo $s'$, $s \neq s'$, with $s'$ being otherwise arbitrary except for $s \neq \hat{0}$. Thus, the assessment of $s$ is in the presence of $s'$, but unless explicitly defined with reference to $s'$, $s$ will be independent of $s'$ in case $s'$ is resolved first with certainty: the persisting risk $s$ constitutes the new status quo and, as such, is no longer constrained by $s'$. Putting $p = s$ in Eqs. (17–19), taking the limits of $\mu_s{}^{crit}$ and $\eta_s{}^{crit}$ for $\omega \to 1$, and considering that $s = s^0$ by definition (cf. Equation (7)), one easily confirms that Eqs. (17) and (18) remain well-defined in these limits,

$$\mu_s^{crit} = \mu_s^{1/2} \tag{23}$$

$$\eta_s^{crit} = \frac{A_s(1+\mu_s^{1/2})}{1-\mu_s^{1/2}} \tag{24}$$

Likewise, Eqs. (19) and (24) approximately agree for $\mu_s \lesssim 1$. Note that, when $s'$ is resolved and the decision maker receives a non-zero outcome, his neutral reference point may shift from $x_0{}' = 0$ in the status quo $s'$ to some $x_0$, $x_0 \neq x_0{}'$, in the new status quo $s$. Subsequently, the outcome axis needs to be rescaled to $x_0 = 0$ to obtain Eqs. (23) and (24).

It remains to clarify the meaning of Eqs. (23) and (24), considering that $\eta_s$ is not defined. Let $q$, $q \in P$, be such that $q$ and $s$ are identical up to $\lambda_q \neq \lambda_s$. In particular, $q^{\pm} = s^{\pm}$ and $q^0 = s$, but $\mu_q \neq \mu_q{}^0 = \mu_s$ and $c_q \neq c_q{}^0 = c_s = 0$. Assume the decision maker to assess $q$ in the status quo $s$, whereby $s$ is resolved first while $q$ persists with certainty. So once more one has $\omega = 1$. With the replacement of $p$ by $q$ in Eqs. (12), (17) and (18), the resulting expressions are well-defined, $\mu_q{}^{crit}$ and $\eta_q{}^{crit}$ exist and, by construction, are equal to $\mu_s{}^{crit}$ and $\eta_s{}^{crit}$, respectively. Since $\eta_q{}^{crit} >> 1$ implies an extreme expected loss $\mu_q{}^- << -1$ if the overall probability of loss $1-\lambda_q$ is small, the same holds true for $\mu_s{}^- << -1$ as a consequence of $\eta_s{}^{crit} >> 1$. Moreover, because $0 < \mu_s < 1$ is supposed, one always has $\mu_s < \mu_s{}^{crit}$, by Eq. (23). Hence, small random perturbations of $s$ yield $\overline{\Delta c_s} < 0$, independently of how small $1-\lambda_s$ is, since such perturbations can be interpreted as probability distributions $q$ with the properties concerned ($\mu_q < \mu_q{}^{crit} = \mu_s{}^{crit}$, $\partial\eta_q/\partial\mu_q < 0$ and $\overline{\Delta c_q} = \overline{\Delta c_s} < 0$). These results make $\eta_s{}^{crit} >> 1$ and $\mu_s < \mu_s{}^{crit}$ meaningful criteria of the catastrophic nature of $s$.

The latter conclusion is of some practical concern. Since according to Eq. (23), for every $\lambda_s$, $\lambda_s < 1$, one has $\mu_s < \mu_s{}^{crit}$, an extreme status quo risk is hard to manage effectively solely by measures to reduce the probability of extreme events. In fact, if $1-\lambda_s$ is lowered while $\mu_s{}^-$ remains constant, $\mu_s$ increases, but so does $\eta_s{}^{crit}$ according to Eq. (24). This means that the robustness of the status quo deteriorates if the overall probability of loss is further reduced while it is already small. There is no paradox involved here, though. The situation corresponds to the conclusion from extreme value theory that small finite tail probabilities of a random variable tend to enhance model uncertainty: the uncertainty in the probability estimate is much larger than the estimated probability value (Huang and Lam 2019). Accordingly, it would seem more appropriate to concentrate on strategies and measures to mitigate the consequences—rather than further reduce the probability—of an extreme event if it is rare anyway. Section 6 outlines effectiveness and efficiency criteria to assess risk management strategies which in this sense are specifically disaster risk-adjusted.

### 4.4 Indicator validity

Since $\eta_p$, $\mu_p{}^{crit}$ and $\eta_p{}^{crit}$ are parameters of simple probability distributions, they can likewise be defined for relative frequency distributions. If so, they quantify attributes not only of random variables, but also of sample data sets. With a slight abuse of notation, by $p$, $q$, $s$, we designate probability and frequency distributions alike. If frequency is intended, they express observed variation rather than uncertainty. Given $x_0$, $s$ and $\omega$, this variation, too, is consistently assessed by the certainty equivalent in preference terms.

Assume that the relative frequency distribution $q$ is obtained by randomly sampling the (unknown) probability distribution $p$ of some variable $X$, with the sample of size $n$, $n \geq 1$. The sampling process needs qualification to count as a source of valid empirical information. "Validity" here means avoidance of systematic error. We first treat the validity of the statistical indicators of extreme risk from a definitional perspective. Operational issues of validation will then be addressed in Subsect. 4.5. Suppose that the sequence of measurements contributing

to $q$ converges to $X$ in distribution for $n \to \infty$.[4] Convergence in distribution of $X$ means that the cumulative probability distribution $F_p(x)$ can be arbitrarily closely approximated by $F_q(x)$ at every continuity point $x$ of $F_p$ by choosing a sufficiently large sample size (as for the conceptual framework of convergence in distribution, see, e. g., Pfeiffer, 1990, pp. 399–407). On these assumptions, the sample parameters ${\mu_q}^{crit}$,$\eta_q$ and ${\eta_q}^{crit}$ converge to the parameters ${\mu_p}^{crit}$,$\eta_p$ and ${\eta_p}^{crit}$ for $n \to \infty$, respectively.

In proof of this convergence, recall that $c_p > 0$, corresponding to $\mu_p > {\mu_p}^0$ in Inequality (10). Hence, each ${\mu_p}^{crit}$,$\eta_p$ and ${\eta_p}^{crit}$ are bounded and continuous functions of the parameters $\mu_p$, ${\mu_p}^0$ and ${\mu_p}^-$, by Eqs. (13), (14), (17) and (18). This conclusion also applies to${\mu_q}^{crit}$, $\eta_q$ and ${\eta_q}^{crit}$ as functions of $\mu_q$, ${\mu_q}^0$, and ${\mu_q}^-$, provided that $\mu_q > {\mu_q}^0$. Since the measurements contributing to $q$ are supposed to converge in distribution to $X$, from the continuous mapping theorem (e. g., Pfeiffer, 1990 p. 406) it trivially follows that the sample averages $\mu_q$, ${\mu_q}^0$, and ${\mu_q}^-$ converge to $\mu_p$, ${\mu_p}^0$, and ${\mu_p}^-$, respectively. Finally, the bounded and continuous dependence of ${\mu_q}^{crit}$,$\eta_q$ and ${\eta_q}^{crit}$ on $\mu_q$, ${\mu_q}^0$, and ${\mu_q}^-$ confirms the above convergence result.

Various basic issues are involved here. First, whether a given sequence of measurements converges in distribution can be tested. Given the finite sample space of $X$, there exist criteria providing—however limited—evidence on convergence in distribution solely and directly with reference to the observed data (Efron, 1979, pp. 22–23). They are based on the resampling methods and Monte Carlo techniques mentioned in the context of the operational validation issues below. Secondly, the information on which risk assessment is based derives from the sample statistics of $q$ in any approximation which can be practically accomplished. Thirdly, if $\eta_q >> 1$, this indicates $c_q$ to vary strongly with $\mu_q$ even if $\mu_q$ changes only marginally, for example, if sampling continues beyond the sample of size $n$ that has been collected up until now. The indication is in the technical sense of Sub Sect. 4.1 and Condition (20). Finally, the change of $\mu_q$ may result not only from a decrease of $\lambda_q$, but also of ${\mu_q}^-$. In fact, if the support of $q$ is a proper subset of the support of $p$ (typically, in case the latter is unknown), ever larger amounts of loss may turn up with non-zero frequencies for increasing $n$ so that ${\mu_q}^-$ decreases further.

#### 4.4.1 Practical significance of statistical indication

We outline the significance of these results by considering a few implications for applied disaster risk analysis. As long as the overall relative frequency of loss remains small, even large losses may fail to caution the observer that still worse outcomes are possible. A potentially misleading assessment of catastrophic risk of this kind may be induced by the measurement of a positive $c_q$. Indeed, $c_q > c_s = 0$ suggests that the risk inherent in $X$ is an improvement over the extant uncertainty or variability $s$. But the improvement comes at a literally disastrous price. This is made evident by a more complete and adequate analysis of the data. Because of $\eta_q >> 1$, there is strong empirical evidence that $p$ is acute: if $\mu_q < {\mu_q}^{crit}$,$c_q$ may fall below zero, and $X$ may take on extreme negative values in future measurements. The case $\mu_q > {\mu_q}^{crit}$ is different. It implies a robust, although still highly left-skewed outcome frequency distribution. This empirical indication of robustness is of practical use where the risk of an extreme event (e. g., outlier, one-shot disaster) cannot be strictly excluded, but

[4] Considering infinite sequences of measurements of random variables does not necessarily lead beyond descriptive statistics. Here, we aim at indicating but not inferring properties of $X$, whereby indication is weaker than inference or estimation, although no less a systematic endeavour. In fact, every parameter employed to indicate disaster risk will here be a finite-sample statistic.

seems sufficiently implausible to be ignored or at least tolerated. The framework of Eqs. (17) and (18) and the condition$\mu_q > \mu_q{}^{crit}$ then leave any decisions to be taken on $p$ data-based and more precisely informed by reason of evidence.

Alternatively, assume that $c_q$ shows a more moderate risk elasticity ($\eta_q \geq \eta_q{}^{crit} \gtrsim 10$, say), and let $X$ change with time.[5] Assume that $X = X'(t)$ is a non-stationary discrete-time process with random variables $X_1 = X'(t_1)$, $X_2 = X'(t_2)$,… at successive times $t_k$, with each $X_k$ being sampled during a sampling period $t_{k+1} - t_k > 0$ beginning at $t_k$, where $k = 1, 2,\ldots$ If successive measurements of $X$ yield frequency distributions $q$ with growing risk elasticities of $c_q$, the measurements of long-term changes of $\eta_q$, $\eta_q{}^{crit}$, etc., may then provide additional information for disaster risk monitoring and planning. For example, more precise and reliable early indications of disastrous climatic changes $X$ (on the time scales of decades) may be obtained from measurements of ongoing global warming (by seasonal sampling of $X$, across different geographical regions, with the individual measurements taken repeatedly over one season beginning at time $t_k$, respectively).

#### 4.4.2 Preliminary application example

To illustrate further the practical significance of these conceptualisations for disaster risk anticipation, we qualitatively refer to a recent salient example. It is the great March 2011 earthquake in what is known in geology as the Japan‘s Northeast (Tohoku) subduction zone. “Many seismologists—and hence disaster planners—thought that such huge earthquakes could not occur on this subduction zone … Great earthquakes—magnitude 8—were expected and planned for. However, a giant magnitude 9 earthquake, which would release 30 times more energy, was not considered. … Thus, the … earthquake generated a huge tsunami that overtopped even 10-m seawalls, causing enormous damage including crippling nuclear power plants” (Stein & Okal, 2011). Why did seismologists and safety planners not anticipate such a huge earthquake on the basis of the available experience? Stein and Okal give three reasons, among others, for this apparent misunderstanding of the historical earthquake record. If these reasons had not been ignored, each could have massively enhanced the severity assessment of future incidents in terms of a large risk elasticity of the assessments. The reasons are errors in estimating the frequency of occurrence ($1-\lambda_p$), relative time scale of risk exposure ($\omega$) and size of damage ($\mu_p{}^-$), respectively: “The apparent pattern resulted from the fact that magnitude 9s are so rare, on average … These are about 10 times rarer than magnitude 8s. Thus, the short seismological record (the seismometer was invented in the 1880s) misled seismologists into assuming that the largest earthquakes known on a particular subduction zone were the largest that would happen” (Stein & Okal, 2011). In contrast, the present risk assessment model proposes that disaster risks tend to exhibit two distinguishing features. First, exposure to disaster risk is indicated, conditional on well-defined risk measurement and assessment criteria, by simple descriptive statistics. Secondly, in the absence of historical parallels, information on presumptive future extremes can, on these criteria, be extracted from the sample parameters and their risk elasticity properties encoded in the historical data. A more subtle account of disaster risk anticipation, which makes these two features visible in detail in the Tohoku earthquake example, is provided in Sect. 5 on the basis of the data given by Kagan and Jackson (2013).

[5] Non-stationary processes and trends are treated, and the relevant literature is reviewed, from various theoretical and application perspectives of extreme value analysis in Visser and Petersen (2012), Visser et al. (2015), and de Haan and Zhou (2021).

### 4.5 Validation issues

In real applications, convergence in distribution of sequences of measurements at best holds approximately, so indication of potential extreme events is susceptible to systematic error in risk measurement. We call models and methods of risk measurement *valid* if they are used in applications and in ways for which they have been designed, methodologically approved (i.e., shown to be logically consistent and correct, unbiased, avoid paradoxical results, etc.) and empirically confirmed. Otherwise, they must be presumed to be sources of systematic violation of the hypotheses to be tested or applied.[6] For example, validation problems of this kind typically arise where "inferences on extremes may be wrong if data are assumed stationary while they are not" (Visser & Petersen, 2012). The problems clearly extend to the indication of extreme risks as well. Of the statistical techniques and methods available to cope with them, we name a few as pervasive examples. They are particularly suited to test the validity of disaster risk indication. Since they are standard techniques of data acquisition and analysis, we will be brief about validation.

Small sample size is a source of random sampling error. But the size of small samples also tends to amplify the effects of these errors systematically (Hox, 2020). It must therefore be subsumed under the validity problems of disaster risk indication, too. Scenario building in combination with software-based simulations of random events, notably Monte Carlo techniques, can help to expand databases to indicate rare extreme events more reliably. The management applications of the present framework suggested below heavily rely on these techniques.

A related basic problem area is the quality and sufficiency of the data available to warrant anticipation of disaster risk. A large risk elasticity of the sample certainty equivalent can be attributed to many different statistical effects such as insignificant outliers, the impact of confounding variables, etc., affecting measurement procedures and scores. The problem is to distinguish whether an indicator measures an attribute of the sampled random variable, some other effect, or a combination of both, which it might be an error to ignore, however (Rice, 2007; Rothman, 2012). We refer to the wide range of data analysis techniques to deal with these and related problems, notably multiple regression, factor analysis and resampling (jackknife, bootstrapping and subsampling; see Politis et al., 1999; Rice, 2007; Boos & Stefanski, 2013). However, the use to be made of these techniques in disaster risk indicator validation is different from the customary uses in hypothesis testing and parameter estimation. If $c_q$ is highly elastic, these techniques can serve to determine whether $\eta_q$ is a resistant statistic (Carling, 2000; Huber & Ronchetti, 2009). This means that resampling techniques can be employed to test whether the observed strong inequality $\eta_q >> 1$ is robust, or "resistant", to the removal of the most extreme negative outcomes from a given sample. The random variable in point reveals its disposition to take on large negative values in this way even if the uncertainty attached to it has not been, or cannot be, entirely resolved.

[6] We tacitly assume that the present concept of validity covers much of what is referred to in the literature as validity and validation of models and methods (National Research Council 2012). Observe that validity depends on the application context. For example, an indicator may prove to be valid when measured in a sampling experiment with replacement, but render the indicated result biased if sampling is without replacement.

## 5 Application examples

### 5.1 Statistical indication of risk of a megathrust earthquake

An illustrative case in point is the statistical record of the seismic activities which gave rise to the unprecedentedly devastating Northeast Japan earthquake and tsunami of March 11, 2011. Seismologists had not foreseen an earthquake of this size (moment magnitude $m \simeq 9.1$) to occur off Japan's Tohoku coast, nor had they believed it to be even possible (Kagan & Jackson, 2013). We refer to the data used in the paper by Kagan and Jackson to demonstrate the present approach in practice. As for the expanding geological research on the hazard potential for very large earthquakes and their recurrence periods, we refer to Schäfer and Wenzel (2019) and Pisarenko and Rodkin (2022) and references given therein. As for analysis and discussion of the geological causes and conditions of the Tohoku megathrust incident, course of events, and types and amounts of damage incurred, we refer to the relevant literature (e. g., Kawase, 2014).

Figure 5 depicts the number $N$ of shallow earthquakes (originating within 0–70 km of the Earth's outer surface) with seismic moment magnitudes equal to or larger than $m$, $5.8 \leq m \lesssim 8.4$, in Japan's Tohoku region during 1977–2010. The sample size is $n = 425$. $N$ is distributed according to the step-like $m$–$N$ curve and approximates the dotted straight line for $m \leq 8$. This line represents the empirical Gutenberg-Richter law. The dashed curves correspond to observed or anticipated departures from the Gutenberg-Richter law for large $m \gtrsim 8$.

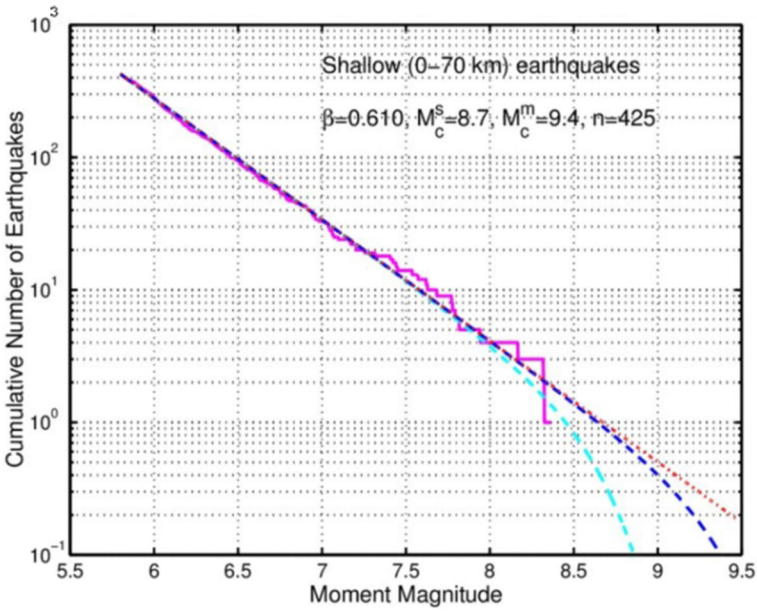



**Fig. 5** Measurement of Earthquake Risk: Example of Seismic Activity in Japan's Tohoku Region. Cumulative Distribution of Number of earthquakes of Moment magnitude ("size") $m$ in the northeastern Japan subduction zone. The data shown were recorded during 1977–2010. The linear part of the curves satisfies the so-called Gutenberg-Richter law. Figure reproduced from Kagan and Jackson (2013)

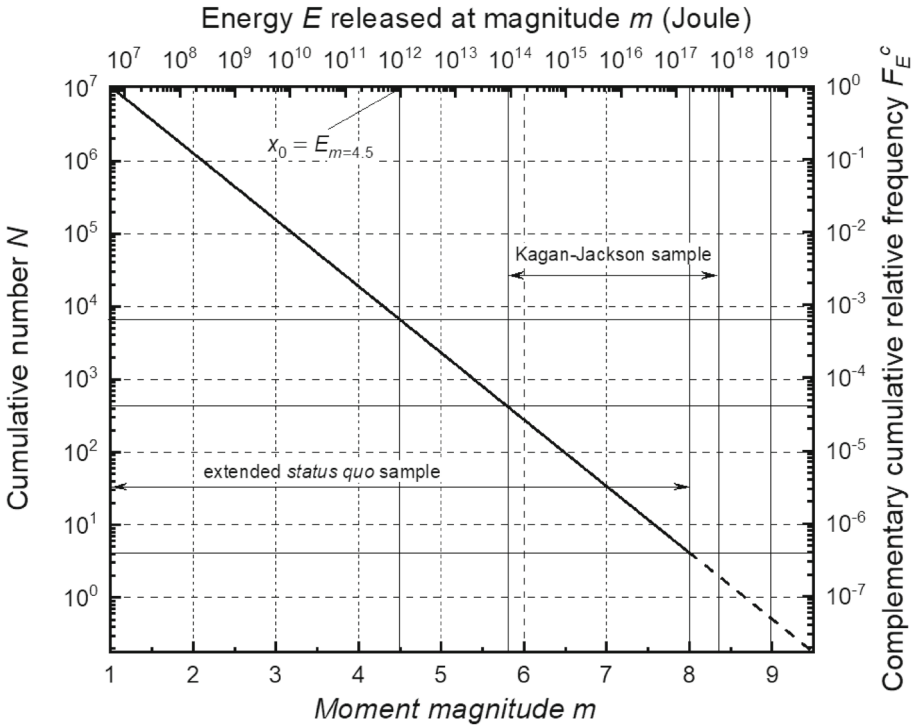



**Fig. 6** Example of Earthquake Risk Expanded from Kagan and Jackson (2013). Extended sample of shallow earthquakes in Japan's Tohoku area. The data for $1 < m < 5.8$ have been extrapolated from the Kagan-Jackson sample using the Gutenberg-Richter law

On the basis of Fig. 5, selected aspects of the status quo risk present in the Tohoku region before March 2011 can be described as follows. The relevant variables, parameters, numerical values and computations are specified and carried out, respectively, in Appendix B. The results are visualised in Fig. 6. The negative value $-E$ of the seismic energy $E$ radiated by an earthquake of magnitude $m$ is measured relative to the amount $E_{m=4.5}$ released at $m = 4.5$ and is defined as the outcome variable $X - x_0 = X = -E + E_{m=4.5}$ in the normalisation $X = x_0 = 0$. This definition corresponds to the notion that "the energy radiated by an earthquake is a measure of the potential for damage to man-made structures" (Spence et al., 1989). Other dimensions of damage such as death toll, injury or economic loss tend to correlate less with $E$ and, for the sake of definiteness, are not made explicit here. Japan's Tohoku region is seismically highly active, so earthquakes and the damage resulting from them are features of everyday life: as damage incurred ($X < 0$, $E > E_{m=4.5}$) for increasingly deleterious and, eventually, devastating consequences if $m > 4.5$; or else as damage avoided ($X > 0$, $E < E_{m=4.5}$) in case of minor events below values of roughly $m \lesssim 4.5$. An approximate magnitude threshold can therefore be conceived as the reference amount of energy released at $m = 4.5$ for which $X = x_0 = 0$. Actually, this threshold means that earthquakes with $m \lesssim 4.5$ tend to be felt less by people and often do not cause appreciable damage to man-made structures—in agreement with the criteria governing the Modified Mercalli scale of earthquake intensity (see, e. g., Lowrie & Fichtner, 2020). Hence the conceptualisation of the case $E < E_{m=4.5}$ as damage avoided, or "gain" (as for the underlying $m$-$E$ relationship, see Appendix B). Meanwhile, the

Kagan-Jackson sample visualised in Fig. 5 has been augmented by data corresponding to $m < 5.8$ and, especially, $m < 4.5$ to incorporate positive outcomes and, thus, define the sample average of gain. We obtained the necessary data missing from the Kagan-Jackson sample by applying the Gutenberg-Richter law to magnitudes as low as $m = 1$ (Schorlemmer et al., 2005, Leary 2020), extending the sample in a consistent, empirically established way. On the other hand, we left out the two or three largest measurements with $m > 8$ from the Kagan-Jackson sample. Removing them from the data set offers a simple "generalised jackknife" measurement and resistance test of the remaining subsample distribution parameters. As for application details on the generalised jackknife methodology, see, for instance, Jekel and Romero (2019). Here, it is employed as a means of backtesting and resampling to demonstrate statistically informed assessments of the hazard looming ahead of an event. The cumulative relative frequency distribution of $E$ is determined by dividing the $m$–$N$ distribution function by the size $N_{m=1} \simeq 10^7$ of the extended sample to obtain relative frequencies for $N$, $E$ and, eventually, $X$. Note that the vertical axes in Fig. 6 are scaled so that $N$ and ${F_E}^c$ give the cumulative number and complementary cumulative relative frequency distributions of the sample, respectively.

Now consider random deviations from ${F_E}^c$ together with the risk elasticities of their certainty equivalents. They can be attributed to subsamples of the extended status quo represented in Fig. 6 (maximum $m = 8$) because deviations with maximum $m > 8$ trivially involve higher damages than $E_{m=8}$ with finite frequencies and, hence, negative certainty equivalents, even if they violate the Gutenberg-Richter law (i. e., deviate from the straight line in Fig. 5). In contrast, subsamples with maximum $m < 8$ have positive $c_m$ by construction, but nonetheless $c_m$ will be unstable if $\eta_m >> 1$ and $\mu_m < {\mu_m}^{crit}$. To show this, we first avoid unnecessarily cumbersome notation by another slight abuse of the formalism: while particular values of the random variables $E$, $N$ and $X$ continue to be written as $E_m$, etc. (or, more explicitly, $E_{m=4.5}$, $N_{m=1}, \ldots$), subscripts are given a different meaning for particular magnitudes $m$ when used to denote sample parameters. In the following, dependence of the statistical parameters on the largest $m$ present in a sample is important, so in expressions like $\mu_m = {\mu_m}^+\lambda_m + {\mu_m}^-(1-\lambda_m)$, subscripts designate the largest $m$ in the sample of which $\mu_m$ is the average. Without loss of generality, these definitions are also meaningful for parameters defined only on subsamples. In particular, one has $\mu_m = {\mu_m}^- = 0$ if $m \leq 4.5$, whereas $\mu_m = {\mu_m}^+ = {\mu_{4.5}}^+ = 1$ and $\lambda_m = \lambda_{4.5} \simeq 1-6{\cdot}10^{-4}$ (Fig. 6) if $4.5 \leq m \leq 8$.

Table 1 gives the critical parameter values for subsamples with different maxima $m$ and one subsample for two different values of $\omega$. In the calculations, it has been assumed that the seismic activities conforming to the Gutenberg-Richter law prevail over long periods of

**Table 1** Disaster Risk Indices for Giant Earthquake (Example of the March 11, 2011 Event in northeastern Japan's seismic zone)

| Sample maximum $m$ of magnitude | $\omega$ | ${\mu_m}^0$ | $\mu_m$ | ${\mu_m}^{crit}$ | ${\eta_m}^{crit}$ | $\eta_m$ |
|---|---|---|---|---|---|---|
| 7.5 | 1 | 0.82 | 0.95 | 0.91 | 34 | 45 |
| 7.8 | 1 | 0.86 | 0.92 | 0.93 | 49 | 50 |
| 7.9 | 1 | 0.87 | 0.91 | 0.93 | 51 | 59 |
| 7.9 | 0.9 | 0.87 | 0.91 | 0.94 | 46 | 55 |
| 7.95 | 1 | 0.89 | 0.9 | 0.94 | 63 | 180 |
| 8 | – | 0.9 ($= \mu_s$) | – | 0.95 | 76 | – |

time, while violations of the law would be rare if any. In this sense, the risk of such violations persists and satisfies $\omega = 1$ in rough, although reasonable, approximation. The results of one more calculation with $\omega = 0.9$ have been added for comparison and in support of this assumption. The table shows increasing values of ${\mu_m}^0$ and ${\mu_m}^{crit}$ from $m = 7.5$ to $m = 8$, that is, risk measurements must yield ever larger positive outcome averages for $c_m$ to be positive and its positive value to be robust ($\mu_m > {\mu_m}^0$ and $\mu_m > {\mu_m}^{crit}$, respectively), whereas the risk of ever larger damage increases (decreasing $\mu_m$) as a function of $m$. Thus, for $m$ well below 8 (maximum $m = 7.5$), the critical risk elasticity is moderately large (${\eta_{7.5}}^{crit} \simeq 34$), and $c_{7.5}$ is robust, that is, it satisfies $\overline{\Delta c_{7.5}} > 0$ (Fig. 7a). At about $m = 7.7$, $c_m$ loses its robustness if $m$ further increases since $\mu_m$ decreases with $m$, while $\overline{\Delta c_m} < 0$. Figure 7b illustrates this situation for $m = 7.8$, with $\mu_{7.8} < {\mu_{7.8}}^{crit}$. For still larger $m$, $\eta_m$ gets exceedingly large ($\eta_{7.95} \simeq 180$), as shown in Fig. 7c, e. Figure 7f illustrates the status quo with maximum $m = 8$, for which ${\eta_8}^{crit}$ is already large, according to Eqs. (23) and (24). The situation remains qualitatively similar if $\omega < 1$ is assumed, as the example of Fig. 7d shows.

Altogether, the present disaster risk indicators admit an utterly novel statistical assessment of the hazard the available data could have provided in advance of the March 2011 event, however. Although the assessment is based on an extended status quo sample with maximum magnitude $m = 8$, the susceptibility of the Tohoku region to extreme earthquakes is already statistically apparent from a subsample with a maximum magnitude of less than 7.8. Even stronger evidence derives from sample maxima $m > 7.8$. They imply a high risk elasticity ($\eta_m >> 1$) and lack of robustness ($\mu_m < {\mu_m}^{crit}$) of the observed risk preference values $c_m > 0$.

It remains the issue of whether and why seismologists and disaster risk planners should have considered a giant magnitude 9 event, who instead "dramatically underestimated" (Kagan & Jackson, 2013) the maximum earthquake size possible on that subduction zone. In contrast, the present account of the situation would have rigorously excluded this underestimate. If the observed sample maxima exceed $m = 8$, the relative frequencies of ever larger earthquakes indeed decline, especially so since the Gutenberg-Richter law tends to be increasingly violated for $m > 8$ (see dashed tapered $m$–$N$ curves in Fig. 5). But at the same time, the observed average damage ${\mu_m}^-$ rapidly decreases and, hence, $\mu_m$ decreases while all the critical parameters involved approach their limits (${\mu_m}^{crit} > {\mu_m}^0 \to 1$, ${\eta_m}^{crit} >> 1$, $\eta_m \to \infty$). They thus indicate a "dramatically" growing instability of both status quo sample distributions and random deviations from the latter. Hence, in the present interpretation, the historical earthquake record could have firmly disclosed, rather than led to underestimate, the risk of magnitude 9 events. From a broader perspective, an upper boundary to moment magnitude can be inferred only from relevant geophysical models and data. More recently, the necessary information has become increasingly available in seismology (Pisarenko & Rodkin, 2022; Schäfer & Wenzel, 2019), but had been partly unknown or attempted to derive from inadequate hypotheses in the years before 2011.

### 5.2 Catastrophic technological risk

In applications of the present formalism to extreme technological risk, the approximation $\omega = 1$ assumed in the Tohoku earthquake example would seem inadequate. Technological systems and devices have limited operational life spans and are as a rule subject to the danger of premature failure. Thus, one typically has $\omega < 1$ in technological risk analyses. The consequences of this situation for the indication and management of technological disaster risk are considerable. If $\omega \leq {\mu_p}^0(1 + {\mu_p}^0)^{-1}$, it follows that ${\mu_p}^{crit} = {\lambda_p}^{crit} = 1$ so that $\mu_p <$

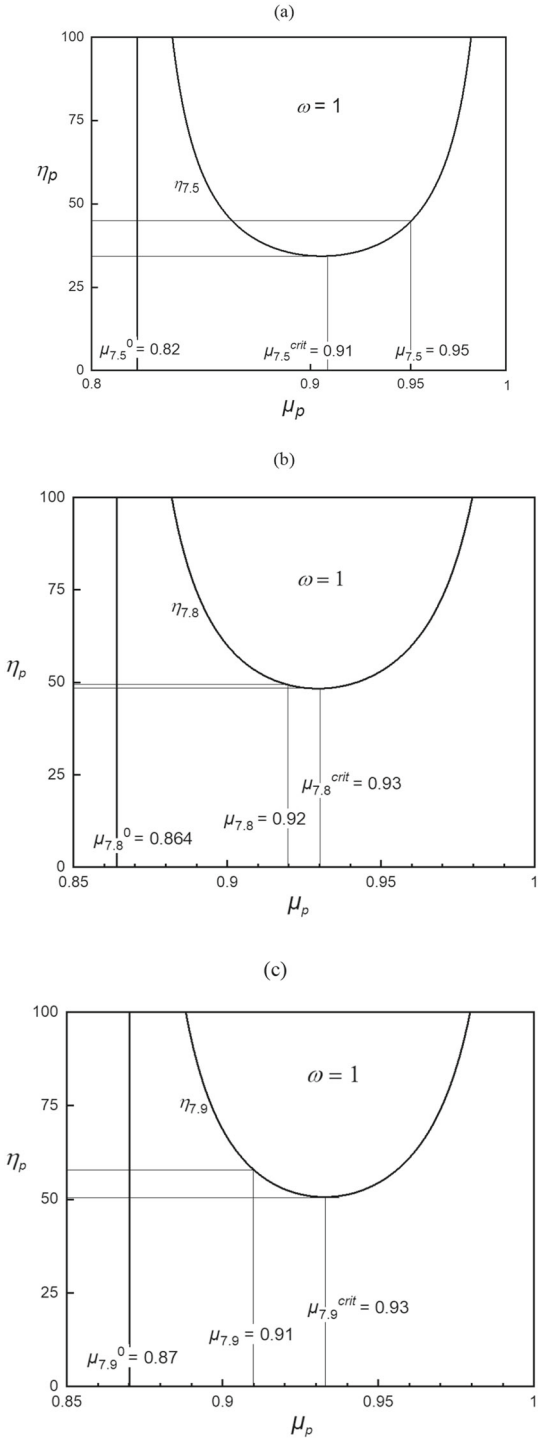
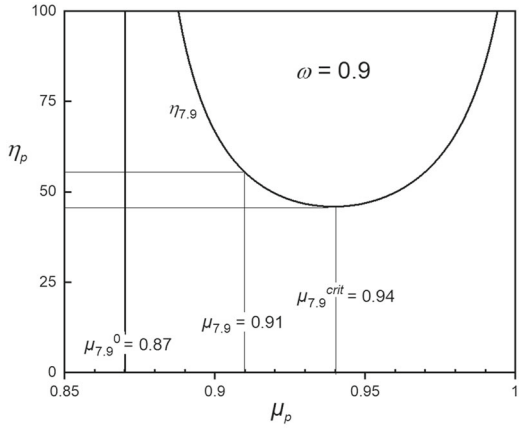


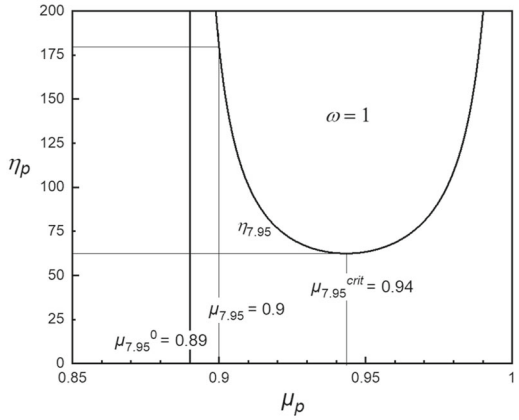

**Fig. 7** Risk Elasticities of Certainty Equivalents of Samples of Earthquakes of Varying Maximum Size $m$. Risk elasticity curves corresponding to ${\eta_m}^{crit}$ as given in Table 1. **a** Moderately large critical risk elasticity for maximum $m$ well below 8 ($m = 7.5$); because of $\mu_{7.5} > {\mu_{7.5}}^{crit}$, $c_{7.5}$ is robust. **b** At about $m = 7.7$, $c_m$ loses its robustness, for example, $\mu_{7.8} < {\mu_{7.8}}^{crit}$. **c**, **e** For still larger $m$, $\eta_m$ gets exceedingly large ($\eta_{7.95} \simeq 180$). **f** Status quo with maximum $m = 8$, for which ${\eta_8}^{crit} = 76$ is already large. **d** Case $m = 7.9$ with $\omega < 1$

**Fig. 7** continued

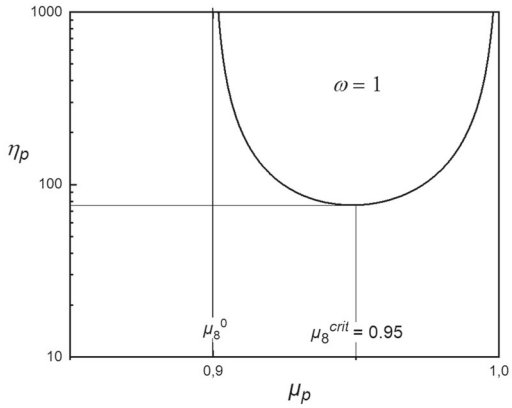
(d)




(e)




(f)



${\mu_p}^{crit}$ and, hence, every catastrophic risk $p$ is acute, even for intermediate $\omega$, however small the overall probability of damage $1-\lambda_p$ may be. If so, $p$ cannot even be made robust by preventive measures reducing $1-\lambda_p$. In fact, $\lambda_p > {\lambda_p}^{crit}$ would have to hold to achieve this, but $\lambda_p$ cannot exceed ${\lambda_p}^{crit} = 1$. This impossibility obtains for all $\omega < 0.5$ since ${\mu_p}^0(1 + {\mu_p}^0)^{-1} \simeq 0.5$ for values of ${\mu_p}^0$ close to unity, on which the present analysis concentrates. To be effective, measures to improve disaster safety would have to reduce system failure rates to ensure values $\omega \geq 0.5$, as has been discussed above with reference to Fig. 4. The following example illustrates this task.

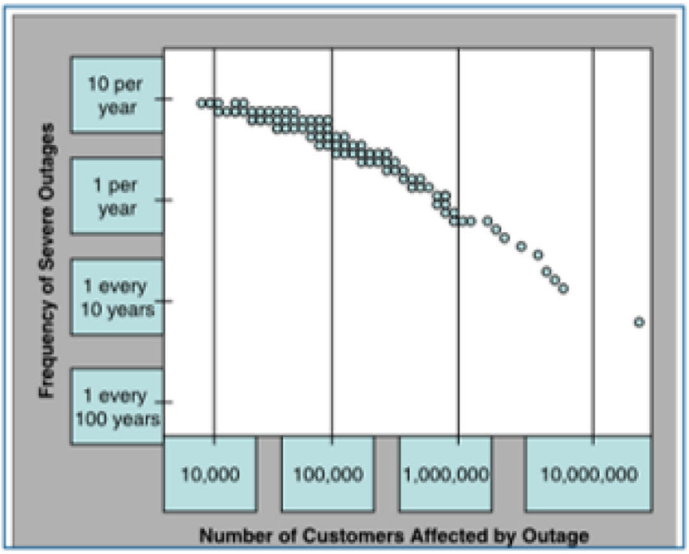
In August 2003, major parts of the Midwest and Northeast of the United States and Ontario, Canada, were affected by an electric power blackout unparalleled in the history of North American power supply systems. According to the Final Report of the U.S.-Canada Power System Outage Task Force (2004), the event affected an area with an estimated 50 million people and 61,800 megawatts of electric load in eight U.S. states and the Canadian province of Ontario. Power was not fully restored for days and, partly, for more than one week in some of these areas. As appears from Fig. 8, shorter and more localised outages occur in North American power systems rather frequently. Long-term disruptions of large-scale power systems are rare, but they occur more frequently than normally distributed outages would do. A statistical representation of major outage events between 1984 and 1997 in North American power systems is given in Fig. 8a by the number of customers affected and the rate of occurrence. The outage of August 2003 corresponds to the extreme event (outlier data point added to the 1984–1997 sample on the lower right side). It surpasses the second-worst incident (August 1996 blackout spreading across Western Canada, the Western United States and Northwestern Mexico) by one order of magnitude in terms of people affected (second outermost data point on the lower right side).

Again, we remove the outermost data point from the sample and examine the remaining subsample for latent extremes. The number $x$ of customers affected is treated as a measure of the amount of damage incurred, in agreement with impact metrics widely used in the literature on power outage analyses (Mukherjee et al. 2018a, b). Figure 8a gives $x_0 \simeq 8000$ as *de minimis* damage regarding major power outages. Correspondingly, we estimate ${\mu_q}^+ \simeq 4000$ in a first approximation because of lack of data for single outages with $0 < x < 8000$. The upper solid curve shown in Fig. 8b is fitted to the scatter plot in Fig. 8a, while the lower curve gives the complementary cumulative relative frequency distribution of the damage incurred in a continuous approximation. The frequency-weighted average from 8000 to 6,000,000 customers affected per outage event was numerically calculated to obtain ${\mu_q}^- = -368{,}170$, with the normalised values ${\mu_q}^+ = 1$ and ${\mu_q}^- \simeq -92$, and with $\lambda_q \simeq 0.999$ and $\mu_q = 0.999 + (1-\lambda_q){\mu_q}^- \simeq 0.91$. Other than in the example of Subsect. 5.1, sufficiently detailed statistical information on the status quo risk is not available from the Final Report, however. So ${\lambda_q}^0$ and ${\mu_q}^0$ are impossible to estimate on this basis. But $\eta_q$ and ${\eta_q}^{crit}$ can be more broadly represented as functions of $\mu_q$ for given, although varying, values of ${\mu_q}^0$ and $\omega$ (Fig. 9).

The figure shows key risk features the affected systems exhibited during the period from 1984 to 1997 and beyond. One such feature is that virtually all values $\eta_q$ are exceedingly high for the sample average $\mu_q \simeq 0.91$. Exceptions only arise for ${\mu_q}^0 = 0.85$ (and below, although not shown). They correspond to a diminishing disaster potential, by decreasing overall amount of damage. The indicator values turn out less exceptional ($\eta_q < 50$) in these cases, but still remain high. In all parameter combinations considered, instability prevails ($\mu_q \simeq 0.91 < {\mu_q}^{crit}$). The case of $\omega$ approaching or falling below 0.5, with ${\mu_q}^{crit}$ approaching 1, is shown in Fig. 9 for constant ${\mu_q}^0 = 0.9$.

The result for $\omega \simeq 0.5$ indicates that, in technological systems, exposure to catastrophic risk need not necessarily be due to unusual circumstances such as extreme weather conditions

(a)




(b)

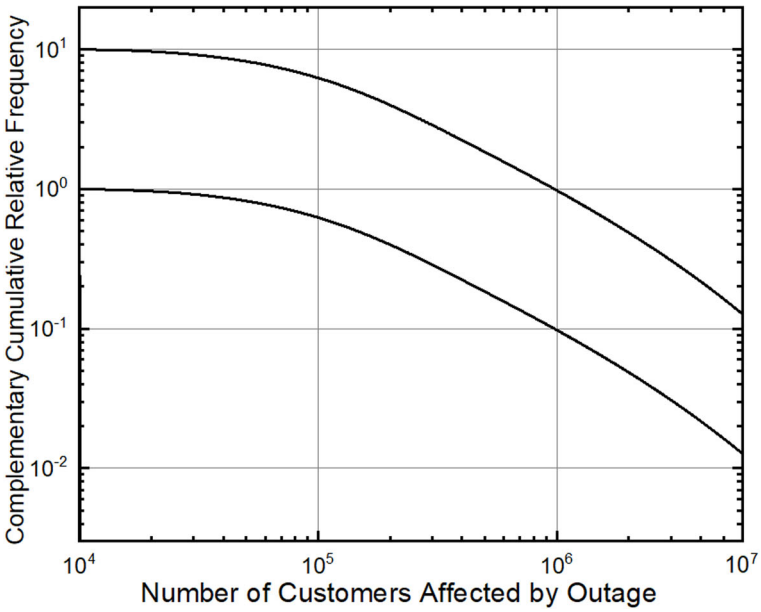



**Fig. 8** North American Power System Outages, 1984–1997. **a** The circles represent individual outages in North America between 1984 and 1997, plotted against the frequency of outages of equal or greater size over that period. Figure reproduced from U.S.-Canada Power System Outage Task Force (2004). **b** Smooth curves fitting the data of (a) (by eye). The upper curve approximates the frequency distribution as scaled in 8(a). The lower curve gives the complementary cumulative relative frequency distribution $q$ in a continuous approximation

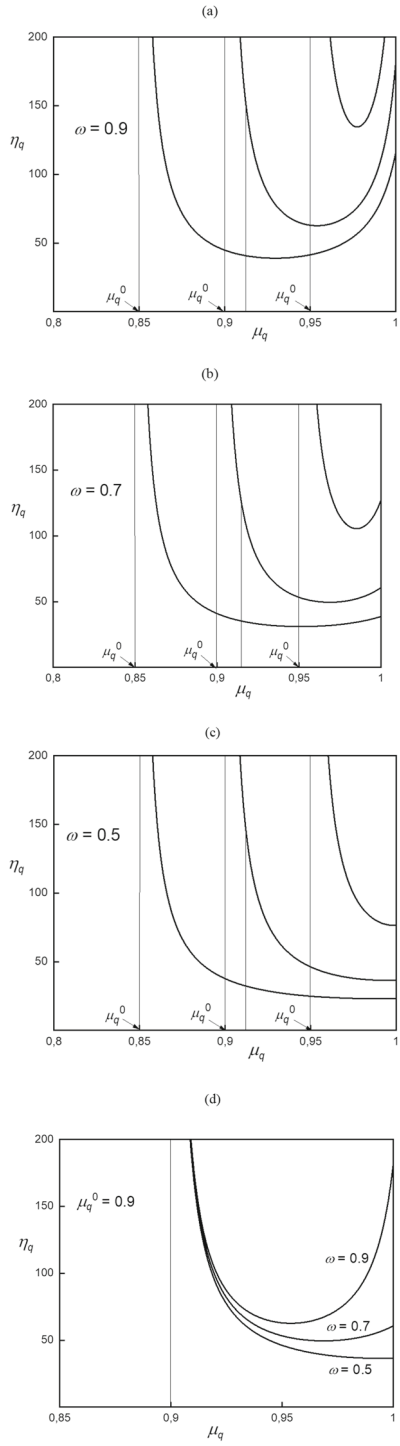



**Fig. 9** Risk Elasticity $\eta_q$ as a Function of $\mu_q$ and $\eta_q{}^{crit}$ for Various Values of $\mu_q{}^0$ and $\omega$. Results for $q$ as shown in Fig. 8b. (**a**–**c**) Risk elasticity $\eta_q$ as a function of $\mu_q$ for varying $\mu_q{}^0$ and $\omega$. **d** Case of $\omega$ approaching 0.5, with $\mu_q{}^{crit}$ approaching 1 for constant $\mu_q{}^0 = 0.9$

or severe lack of maintenance. Catastrophic risk may well arise under normal operating conditions, which will often also constitute the status quo of a system and, as such, involve equipment failure, cascading disruptive events, and so on. Then, the failure of status quo operations and severe disruptions of a system may be rare, but the relative persistence of either type of failure risk is about equally probable, for failures of both types draw on the same source of initiating events (Morgan et al., 2011). This kind of latent extreme risk can in principle be indicated and assessed systematically once $\omega$ can be determined or estimated with some confidence in a probabilistic risk analysis.

## 6 Disaster risk management

### 6.1 Effectiveness and cost-efficiency of risk management

In the presence of catastrophic threats to human safety and security, risk reduction and the effectiveness with which it can be accomplished are primary concerns of disaster risk management. These topics have been treated in the literature from various economic, although not necessarily utility-oriented, perspectives of cost–benefit analysis (Layard & Glaister, 1994; Shreve and Kelman 2014; Fischhoff 2015; Mechler, 2016; Mishan and Quah 2020). We restrict the present account to a few basic aspects to demonstrate its potentials for the practice of disaster risk management.

Risk management decisions can normally be carried out with larger or smaller loss prevention effort and success, leading to a lower ($p$) or higher ($q$) persistent residual risk. Accordingly, $c_p \geq c_q \geq c_s = 0$ gives different degrees of effectiveness of the safety measures planned or taken. Conversely, if $c_s = 0 > c_q$, one may still have $0 > c_p > c_q$. In this case, the risk management procedure (to be) taken proves ineffective or insufficient: neither choice $p$ nor $q$ will reduce the risk the decision maker is currently facing although $p$ means a risk reduction over $q$. Let $k(p, q)$ be the expenditure for some particular measure to reduce a risk $q$ to the "smaller" risk $p$ the risk planner or decision maker prefers. Then, the ratio $(c_p - c_q)/k(p, q)$ gives the amount of risk reduction achieved per unit of money invested, that is, the cost-efficiency of risk reduction. These concepts of risk reduction performance dispense with comparisons between the cost incurred and loss avoided. Comparisons of this kind are central to cost–benefit approaches to risk analysis, notably monetisations of the value of human life. But they are also known to be difficult to conceptualise since they must be based on trade-offs between quantities which are actual (cost) on the one hand, and virtual (benefit conceived as loss avoided) on the other hand. In contrast, the present concept of cost-efficiency—correctly—admits comparison only between actual, although uncertain, outcomes due to $p$ and $q$ ranked on a common risk preference scale. The amount of risk reduction achieved is thus defined and assessed independently of the reduction cost $k(p, q)$, as it should.

On the other hand, effectiveness of risk reduction is only a minimum requirement for the adequacy of disaster risk management choices. The following definitions provide a broader, disaster risk-adjusted perspective on managerial decisions. Let once more be $s$ a status quo risk such that ${\eta_s}^{crit} >> 1$ according to Eq. (24), and define

$$\xi_p = \frac{\mu_p - \mu_p^{crit}}{1 - \mu_p^{crit}}, \mu_p^0 \leq \mu_p \leq 1 \tag{25}$$

Risk management actions transforming $s$ into a less risky residual state $p$ are called *disaster risk-effective* if $0 \leq \xi_p \leq 1$, and *disaster risk-ineffective* if $\xi_p < 0$. These definitions are only partly analogous to those of the criteria of effectiveness of risk reduction ($c_p \geq 0$) because $\xi_p = 0$ corresponds to $\mu_p = \mu_p{}^{crit}$ but not to $\mu_p = \mu_p{}^0$. In this sense, disaster risk-effectiveness means more than that the preferred risk management provisions engender risk reduction from $s$ to some $p$ with $c_p \geq 0$. It requires an at least marginally robust residual situation $\mu_p \geq \mu_p{}^{crit}$, or $\xi_p \geq 0$. Robustness can be improved by measures to decrease the probability of damage (ideally, $\mu_p \to 1$ so that perfect disaster risk-effectiveness $\xi_p = 1$ is approached); mitigate the expected damage (reduce $-\mu_p{}^-$ so that $\eta_p$ decreases, that is, $\eta_p \to 1$ because of $\partial\mu_p{}^0/\partial\mu_p{}^- < 0$ according to Equation (A.5) and $\partial\eta_p/\partial\mu_p{}^0 > 0$ from Eq. (13)); or suitably manipulate the exogenous constraints on risk management such as the relative persistence of risk ($\omega \gtrsim 0.5$), as has been discussed above (see also next subsection). On the other hand, risk management provisions designed to reduce $s$ to $p$ can be effective in the sense of $c_p \geq 0$, while not being disaster risk-effective ($\xi_p < 0$ because of $\mu_p{}^0 < \mu_p < \mu_p{}^{crit}$). Assessments of the effectiveness of risk management based on Eq. (25) are thus suitable to identify and help to avoid insufficient disaster risk reduction practices. A similar point can be made regarding the frequently observed discrepancy between what is effective and desirable, and what is cost-efficient and affordable, in disaster risk management—for instance, in view of the tremendous cost of global climate disaster prevention. This discrepancy and related problems of what is desirable and what is affordable in disaster risk management can be made accessible to quantitative analysis and application by defining the cost-efficiency of disaster risk reduction as $(\xi_p - \xi_q)/k(p, q)$ for choices $p$ and $q$ with $\xi_p \geq \xi_q \geq 0$. Meanwhile, to compare $p$ and $q$ in terms of $\xi_p$ and $\xi_q$, the equation $\mu_p{}^{crit} = \mu_q{}^{crit}$ need not necessarily hold. It is sufficient to assume (irrespectively of the use of normalised, dimensionless $\mu_p$, $\mu_p{}^{\pm}$, etc.) that the outcomes of $p$ and $q$ are measured on a common scale. Otherwise, extensions of the approach to multi-attribute disaster risk assessment along the lines established in Geiger (2012, 2015) are required.

### 6.2 Disaster risk planning

The performance of disaster risk management procedures can, in principle, be measured, tested and assessed within the same methodological framework as that outlined in Sub-Sects. 4.4 and 4.5. However, management uses of this framework must also confront risks conceived in the planning process but never before experienced in practice. Because of the inherent lack of historical data on newly created risks, this problem is particularly critical in risk prevention planning, and in the design and appropriate dimensioning of disaster risk safeguards. In many applications, future uncertainty can be narrowed down, in principle, using physical and economic models and the simulation of uncertain events and processes in scenario- and software-based stochastic experiments under varying conditions. The relevant methodologies are available from model building in the empirical and operational sciences, what-if scenarios, Monte Carlo algorithms, and a wide range of sampling and data analysis techniques. The contribution of the present approach to disaster risk management is then to take the risk attributes together with their simulated variability as surrogate random variables and rank order them according to uniquely and coherently defined, quantitative preferences and risk elasticity properties. They can thus serve to guide planning and decision making in thoroughly information-based, disaster risk-effective (as against more heuristic and ad hoc) ways.

### 6.3 Disaster return periods

Return periods are time intervals within which recurrent (natural hazard) events of a particular kind and intensity occur once on the average. Thus, the "100-year flood" in a river catchment area by definition has a recurrence period of 100 years, meaning that flood events of equal or exceeding flood level occur with relative annual frequency of 1% in that area. Return periods are important for the management of natural hazards since they help to benchmark the size and design of protective systems in prevention planning. Extreme event return periods can be given a probabilistic account in terms of exceedances of random variables above a pre-selected threshold ("peaks over threshold", POT) (Beirlant et al., 2004; Falk et al., 2011).

The present framework of disaster risk analysis offers a different approach to the concept of mean time to resolution of risk. It remains entirely within the scope of established approaches to statistical survival time analysis, most obviously to those using non-parametric methods and bivariate, discrete-time models (Kalbfleisch & Prentice, 2002; Klein et al., 2014; Marshall & Olkin, 2007). As for more detailed consequences of this shift of perspective away from POT, see Read and Vogel (2016), and Vogel and Castellarin (2017). Here, we briefly indicate how the concept of relative persistence of a risk $p$ given $s$ makes the basic idea of the dependence of risk assessment on time-to-event explicit. Consider the simple case of constant hazard rates $\tau_p$ and $\tau_s$ associated with $p$ and $s$, and stochastic independence of $X$ and $T_p$, and $Y$ and $T_s$, where $X$ and $Y$ are the outcome variables, and $T_p$ and $T_s$ the resolution times of $p$ and $s$, respectively (see Sect. 2). Then, for large sample size, one approximately has $\omega = \tau_s(\tau_p + \tau_s)^{-1}$. The average return period of events $X = x$ is ${\tau_p}^{-1} = \omega {\tau_s}^{-1}(1-\omega)^{-1}$. It is large if $p$ is highly persistent compared to the lifetime of $s$ ($\omega \lesssim 1$), and small in case $\omega << 1$: given $s$, larger recurrence intervals of $p$ leave disaster risks more robust since they correspond to larger $\omega$, consistently with the findings above.

Several points are to be noted here. Although $X$ and $T$ are assumed independent, preferences for $X$ generally do depend on $T$. This dependence is an important determinant of disaster risk indication. As such, it contributes to distinguishing risks $p$ with $\eta_p \geq 1$ from those with $\eta_p >> 1$, and disaster risks with $\mu_p$ below from those above ${\mu_p}^{crit}$. Finally, established approaches to the sampling and analysis of lifetime data are specifically suitable to support the statistical indication and, thus, effective management of disaster risk.

### 6.4 Regulatory standards for safety and security

Similar points can be made regarding legal, administrative and other socio-political standards of disaster risk management. Although the current approach has been conceptualised with reference to individuals' optimal risky choices, it can equally be employed for regulatory purposes such as the setting of risk tolerance standards. Applications of this kind may not seem obvious since administrative risk regulation is guided (albeit not exclusively, see Bowen & Panagiotopoulos, 2020) by the precautionary principle rather than principles of optimal choice (de Sadeleer, 2006; Fisher, 2007). But introducing $x_0$ as an aspiration level amounts to setting standards of positive and negative consequences for risk management decisions. In this sense, $x_0$ may represent, or be calculated from, diverse reference levels such as the maximum allowable effective radiation dose or dose rate per person specified by a radiological regulatory agency; regulated price and return indices in industries shaped by natural monopolies ("price-cap" and "rate-of-return" regulation); or a maximum amount of admissible $CO_2$ emissions set up by an international binding agreement to mitigate global climate change. Consistently with any such definition of $x_0$, all further steps to test and assess disaster risk-effective

management procedures can, in principle, proceed as outlined above. Since aspiration or tolerance levels laid down by regulation are often subject to public controversy, the tests can especially help to clarify reason and significance of the disputed levels in information-based and explicitly risk-adjusted ways.

## 7 Conclusions

The preceding analysis draws largely on the explicit, simple algebraic representation the certainty equivalent $c_p$ of a random variable $X$ with probability distribution $p$ has been given within the present non-expected utility framework. In this representation, the risk elasticity $\eta_p$ of $c_p$ has been introduced as a characteristic parameter which, for values $\eta_p >> 1$, indicates whether large random samples of $X$ will include rare outcomes $X = x << -1$. The risk elasticity parameter can also be defined as a sample statistic and, as such, directly computed from the measured data. Then, the empirical indication that $X$ is a catastrophic risk takes on the form $\eta_q >> 1$, where $q$ is the sample distribution of a sufficiently large number of randomly drawn measurements of $X$. The indicative power of sample values $\eta_q >> 1$ further rests on the ability of the approach to distinguish between measurements $\eta_q$ which show $c_q$ to be stable ("robust") or else unstable to small random perturbations of $X$. If instability prevails, random variations of $X$ even of limited size (outcome value and probability) will lead to large changes of $c_q$ on continued sampling, indicating extreme events $X = x << -1$ with finite, although unknown, probability. To obtain these results, probabilistic modelling of extreme events is not required.

In real applications of the approach, various limits of applicability will likely become apparent. Often, the data required for disaster risk analysis are too incomplete or strongly biased for the risks in point to be captured by the present framework. Moreover, often the most informative data are accessible only in aggregate form such as annual averages of loss or damage published by administrations or the insurance industry. As mentioned above, there are numerous methodologies available in modern risk science to cope with these shortcomings, however. Basically, these are modelling and scenario building of the physical and social systems and processes at risk, software-based simulation and data processing techniques, Monte Carlo, resampling and backtesting approaches in applied statistics, and combinations of these. The present explicit, quantitative and, after all, realistic concept of risk assessment contributes to exploiting more effectively these methodologies for disaster risk analysis and management. Sections 5 and 6 take steps in this direction.

A similar point can be made regarding catastrophic security risks. In contrast to natural hazards, for instance, security risks tend to involve planned, purposeful human action (e.g. terrorist attacks). As such, they are quite the opposite of random events and ought to be modelled by "games of strategy" rather than "games of chance" (von Neumann & Morgenstern, 1947). Alternatively, security incidents can be conceptualised as what-if scenarios, with disaster risk analysis and assessment concentrating on their probabilistic damage consequences. Scenario building is, thus, taken as a source of reference cases relative to which the probabilistic attributes of security incidents can be analysed in definite ways. What-if scenarios in combination with the present risk assessment model can accordingly serve as planning instruments in catastrophic security risk indication and prevention planning.

Another field of applicability of the approach can be envisaged in management applications to risky choice with multi-dimensional attributes of risk and non-expected utility trade-offs between them (Geiger, 2012, 2015). A similar point can be made regarding collective decision

making with potentially disastrous consequences. The advantage offered by the approach in these applications would be that, to apply the present conceptual framework, one only has to specify the exogenous parameters $s$, $x_0$ and $\omega$ appropriately. The specification will generally be independent of utility (see, e. g., Geiger, 2012 in the multi-attribute utility case), whereas the subsequent disaster risk indication and risk assessment would proceed as described above.

## 8 Appendix A Derivations of formulae

### 8.1 Derivation of Eqs. (13) and (14)

With the use of Eq. (5), rewrite Eq. (4) as

$$c_p = \frac{(1-\omega)(\mu_p - \mu_p^0)}{\omega(1-\mu_p) + (1-\omega)(1-\mu_p^0)}, \mu_p \geq \mu_p^0$$

where the normalisations $\mu_p^+ = \left(\mu_p^0\right)^+ = 1$ and $\left(\mu_p^0\right)^- = \mu_p^-$ are once more adopted, $\lambda_p$ is replaced by $(\mu_p - \mu_p^-)/(1-\mu_p^-)$, and similarly for ${\lambda_p}^0$. Observe that $c_p$ explicitly depends on $\mu_p$ and ${\mu_p}^0$, but ${\mu_p}^0$ implicitly depends on $\mu_p$, that is, on ${\mu_p}^-$, which in turn determines ${\lambda_p}^0$ from Eq. (2) and $\mu_p^0 = \lambda_p^0 + \mu_p^-(1-\lambda_p^0)$. Hence,

$$\frac{\mathrm{d}c_p}{\mathrm{d}\mu_p} = \frac{\partial c_p}{\partial \mu_p} + \frac{\partial c_p}{\partial \mu_p^0} \cdot \frac{\mathrm{d}\mu_p^0}{\mathrm{d}\mu_p} \tag{A.1}$$

One straightforwardly finds

$$\frac{\partial c_p}{\partial \mu_p} = \frac{(1-\omega)(1-\mu_p^0)}{(-\omega(\mu_p - \mu_p^0) + 1 - \mu_p^0)^2} \tag{A.2}$$

$$\frac{\partial c_p}{\partial \mu_p^0} = \frac{(1-\omega)(1-\mu_p)}{(-\omega(\mu_p - \mu_p^0) + 1 - \mu_p^0)^2} \tag{A.3}$$

Trivially, one has $\mathrm{d}\mu_p/\mathrm{d}\mu_p^- = \partial\mu_p/\partial\mu_p^-$ and $\mathrm{d}\mu_p^-/\mathrm{d}\mu_p = 1/\left(\mathrm{d}\mu_p/\mathrm{d}\mu_p^-\right)$. Hence, $\mathrm{d}\mu_p^-/\mathrm{d}\mu_p = (1-\mu_p^-)/(1-\mu_p)$, which together with

$$\frac{\mathrm{d}\lambda_p^0}{\mathrm{d}\mu_p} = \frac{1}{\mathrm{d}\mu_p/\mathrm{d}\mu_p^-} \cdot \frac{\mathrm{d}\lambda_p^0}{\mathrm{d}\mu_p^-} \tag{A.4}$$

gives

$$\begin{aligned} \frac{\mathrm{d}\mu_p^0}{\mathrm{d}\mu_p} &= \frac{\partial\mu_p^0}{\partial\mu_p^-} \cdot \frac{\mathrm{d}\mu_p^-}{\mathrm{d}\mu_p} + \frac{\partial\mu_p^0}{\partial\lambda_p^0} \cdot \frac{\mathrm{d}\lambda_p^0}{\mathrm{d}\mu_p} \\ &= \frac{1-\mu_p^0}{1-\mu_p} + \frac{(1-\mu_p^-)^2}{1-\mu_p} \cdot \frac{\mathrm{d}\lambda_p^0}{\mathrm{d}\mu_p^-} \end{aligned} \tag{A.4}$$

The probability ${\lambda_p}^0$ is defined as an implicit function of ${\mu_p}^-$ by Eq. (2), which we rewrite as $\mu_p^0 = z_s(\mu_p^0 - \mu_p^-)^{1/2}(1-\mu_p^0)^{1/2}$. From this, we compute $\mathrm{d}\mu_p^0/\mathrm{d}\mu_p^-$ by means of the

implicit function theorem and insert the result

$$\frac{\mathrm{d}\mu_p^0}{\mathrm{d}\mu_p^-} = -\frac{\mu_p^0(1-\mu_p^0)}{\mu_p^0-\mu_p^-(2-\mu_p^0)} \tag{A.5}$$

into

$$\frac{\mathrm{d}\lambda_p^0}{\mathrm{d}\mu_p^-} = \frac{\mathrm{d}}{\mathrm{d}\mu_p^-}\frac{\mu_p^0-\mu_p^-}{1-\mu_p^-}$$

to obtain

$$\frac{\mathrm{d}\lambda_p^0}{\mathrm{d}\mu_p^-} = \frac{2(1-\mu_p^0)(\mu_p^0-\mu_p^-)}{(\mu_p^0-\mu_p^-(2-\mu_p^0))(1-\mu_p^-)^2} \tag{A.6}$$

Inserting Equation (A.6) into Equation (A.4) and the result together with Equations (A.2) and (A.3), into Equation (A.1), eventually gives

$$\frac{\mathrm{d}c_p}{\mathrm{d}\mu_p} = \frac{2(\mu_p^0-\mu_p^-)}{\mu_p^0(1+\mu_p^-)-2\mu_p^-}\cdot\frac{(1-\omega)(1-\mu_p^0)}{(-\omega(\mu_p-\mu_p^0)\ +1-\mu_p^0)^2}$$

Multiplication of the latter expression with

$$\frac{\mu_p}{c_p} = \frac{\mu_p(-\omega(\mu_p-\mu_p^0)\ +1-\mu_p^0)}{(1-\omega)(\mu_p-\mu_p^0)}$$

entails Eqs. (13) and (14).

### 8.2 Derivation of Eq. (22)

Half of the disturbances of $\mu_p$ are positive and supposed to sum up to a total of $\Delta\mu_p$, the other half are negative and altogether equal to $-\Delta\mu_p$, where $\Delta\mu_p$ is positive and $\Delta\mu_p/\mu_p$ is small. For the perturbations $\Delta c_p$ due to $\Delta\mu_p$, one has

$$\Delta c_p \simeq \Delta\mu_p \mathrm{d}c_p/\mathrm{d}\mu_p = c_p\eta_p\mu_p^{-1}\Delta\mu_p$$

and

$$-\Delta c_p' \simeq -\Delta\mu_p\left(\mathrm{d}c_p/\mathrm{d}\mu_p\right)' = -c_p'\eta_p'(\mu_p')^{-1}\Delta\mu_p$$

for negative perturbations with $\mu_p{}' = \mu_p - \Delta\mu_p$, where $c_p{}' = c_p + \Delta c_p{}' = c_p - \Delta\mu_p(\mathrm{d}c_p/\mathrm{d}\mu_p)'$, etc. As supposed in Eq. (16), the perturbations do not depend on $\mu_p{}^-$ and $\mu_p{}^0$. Hence, $(\mathrm{d}\eta_p/\mathrm{d}\mu_p)'$ equals $(\partial\eta_p/\partial\mu_p)'$ so that

$$\left(\partial\eta_p/\partial\mu_p\right)' = \partial\eta_p/\partial\mu_p - \Delta\mu_p\partial^2\eta_p/\partial\mu_p^2$$

Expanding $(\mu_p{}')^{-1} = (\mu_p - \Delta\mu_p)^{-1}$ in the equation for $\Delta c_p{}'$ into $(1 + \mu_p{}^{-1}\Delta\mu_p)\mu_p{}^{-1}$, putting $\overline{\Delta c_p} = \Delta c_p + \Delta c_p'$ and neglecting all but the lowest-order perturbation terms, one immediately gets the result Eq. (22).

## 9 Appendix B Calculation of entries in Table 1

At first, the sample parameters to be listed in Table 1 are computed for the variable $E$ as the relevant measureable quantity. The values obtained in this way are then rescaled in a separate step to make them compatible with the change of variables and normalisations adopted elsewhere in the analysis,

$$E \to X - x_0 = -E + E_{m=4.5}, x_0 = 0$$

and

$$\mu_m^+ \to \mu_m^+ = 1.$$

We use the Gutenberg–Richter law to represent the $m$–$N$ relationship,

$$\lg(N) = a - bm$$

where "lg" denotes the logarithm to the base 10, and $a$ and $b$ are parameters which take on special values for any seismic zone. Kagan and Jackson find $b = 1.5\beta$, with $\beta = 0.61$ for the Tohoku area, and $a = 7.9$ from $N_{m=5.8} - 2 = 423$. When measured in units of Joule, $E$ is a function of $m$ approximately given by $\lg(E) = 5.24 + 1.5m$. Thus, $N = 10^{11.1}E^{-\beta}$. Rewriting this as $N_m = 10^{11.1}{E_m}^{-\beta} = N_E$ and considering $N_{m=1} \simeq 10^7$, one gets

$$F_E^c = N_E/N_{m=1} = 10^{4.1}E^{-\beta}$$

as the complementary cumulative relative frequency distribution of the random variable $E$ in a continuous approximation (Fig. 6), and $f_E = 10^{4.1}\beta E^{-1-\beta}$ for the approximate relative frequency distribution. Using the denotation introduced in Subsect. 5.1, one has $\lambda_m = \lambda_{4.5} \simeq 1 - 6{\cdot}10^{-4} \simeq 1$, with the neutral reference point $E_{m=4.5}$ and, hence, $\lambda_m = F_{E_{4.5}}$ being invariant. Before the normalisations assumed in Sects. 2 and 3 are carried out, one now determines ${\mu_m}^{\pm}$ conveniently in continuous approximation,

$$\mu_m^+ = 10^{4.1}\beta\lambda_m^{-1}\int_{E_{m=1}}^{E_{m=4.5}} E^{-\beta}\mathrm{d}E = 10^{4.1}\beta(1-\beta)^{-1}\lambda_{4.5}^{-1}(E_{m=4.5}^{1-\beta} - E_{m=1}^{1-\beta})$$

$$\mu_m^- = 10^{4.1}\beta(1-\lambda_m)^{-1}\int_{E_{m=4.5}}^{E_m} E^{-b}\mathrm{d}E = 10^{4.1}\beta(1-\beta)^{-1}(1-\lambda_{4.5})^{-1}(E_m^{1-\beta} - E_{m=4.5}^{1-\beta})$$

$$\mu_m = \mu_m^+\lambda_m + \mu_m^-(1-\lambda_m) = 10^{4.1}\beta(1-\beta)^{-1}(E_m^{1-\beta} - E_{m=1}^{1-\beta})$$
$$\simeq 10^{4.1}\beta(1-\beta)^{-1}E_m^{1-\beta}$$

Thereby, the final approximation for $\mu_m$ follows from $E_m >> E_{m=1}$ for $7.5 \le m \le 8$ (see Fig. 6). Carrying out the normalisations chosen, one finally gets,

$$\mu_m^{\pm} \to \frac{-\mu_m^{\pm} + E_{m=4.5}}{-\mu_m^{+} + E_{m=4.5}}, E_{m=4.5} = 10^{12}$$

To complete the calculations behind Table 1, ${\mu_m}^0$ and, as a prerequisite, the parameters of the status quo $s$ entering Eq. (2) must also be determined. The "extended status quo sample" is as shown in Fig. 6, with maximum $m = 8$. The only nontrivial parameter is ${\mu_8}^- = -184$

in the final normalisation, the other parameter values entering the left-hand side of Eq. (2) are as above ($\lambda_8 = \lambda_{4.5} \simeq 1 - 6{\cdot}10^{-4} \simeq 1$, ${\mu_8}^+ = 1$),

$$z_8 = \frac{1-6 \cdot 10^{-4} \cdot 184}{10^{-2}(1+184)\sqrt{6}} \simeq 0.2$$

so that

$$0.2 = \frac{\lambda_m^0 + \mu_m^-(1-\lambda_m^0)}{\left(1-\mu_m^-\right)\sqrt{\lambda_m^0(1-\lambda_m^0)}}$$

can be solved for ${\lambda_m}^0$, given ${\mu_m}^-$ as above. Now, ${\mu_m}^0 = {\lambda_m}^0 + {\mu_m}^-(1 - {\lambda_m}^0)$ is completely determined. The remaining quantities $\eta_m$, ${\mu_m}^{crit}$ and ${\eta_m}^{crit}$ can be calculated according to Eqs. (13) and (14), (17) and (18), respectively. These steps complete the specification of Table 1.

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## Declarations

**Conflict of interest** Author Gebhard Geiger declares that he has no conflict of interest.

**Ethical approval** This article does not contain any studies with human participants or animals performed by the author.

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